Quiz I, October 30, 2004 **Solution**

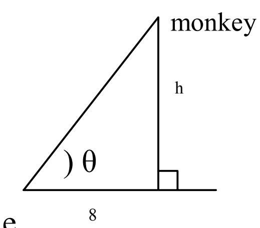
1. The derivative of $x \sin^{-1}x$, by the product rule, is clearly

$$\sin^{-1}x + \frac{x}{\sqrt{1-x^2}}$$

Note that another notation for $\sin^{-1}x$ is $\arcsin x$.

2. The height h of the monkey and the angle θ of observation are both functions of time t. Their relationship is given by

$$\tan\theta = \frac{h}{8}.$$



We are given that $\frac{dh}{dt} = 7.5$ ft/s and want to find $\frac{d\theta}{dt}$ when h = 16. Applying D_t to the above equation gives, by chain rule

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{8}\frac{dh}{dt} = \frac{1}{8} \times 7.5$$

At h = 16 the longest side of the triangle equals $\sqrt{8^2+16^2}$, for which $\sec\theta$ equals $\frac{\sqrt{8^2+16^2}}{8} = \sqrt{5}$. So $(\sqrt{5})^2 \frac{d\theta}{dt} = \frac{1}{8} \times 7.5$ and $\frac{d\theta}{dt} = \frac{3}{16}$ rad/s **(ANSWER)**

3. $y = f(x) = \frac{e^{3x} \cos x}{\sqrt{4+x^2}}$, In logarithmic differentiation the first step is to

" take ℓn of both sides " :

$$\ell n\, y = \ell n\, e^{3x} + \ell n \cos x - \ell n \sqrt{4+x^2}, \text{ or}$$

$$\ell n\, y = 3x + \ell n \operatorname{cox} x - \frac{1}{2} \ell n\, (4+x^2).$$

Implicit differentiation of the last equation with respect to x gives

$$\frac{1}{y} y' = 3 + \frac{1}{\cos x}(-\sin x) - \frac{1}{2}\frac{1}{4+x^2}(2x).$$

Substituting x = 0, and noting $y(0) = \frac{1}{2}$, we get

$$2y'(0) = 3 + 0 - 0 = 3. \quad \text{Hence } y'(0) = \frac{3}{2}$$ **(ANSWER)**

4. We want to find the slope of the tangent line to the curve $x^3 + xy + y^2 = 7$ at the point (1,2). First, by a routine check, this point does indeed lie on the curve.

Applying D_x as in implicit differentiation, we obtain

$$3x^2 + \left(y + x \frac{dy}{dx} \right) + 2y \frac{dy}{dx} = 0.$$

Substitution x = 1, y = 2 gives

$5 + 5 \frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -1$ at the point (1,2). This is the required slope.

5. Let y be the coffee temperature at time t with t = 0 corresponding to 1:00 p.m. By Newton's law of cooling, y satisfies the Differential Equation

$$\boxed{\frac{dy}{dt} = k\,(y-70),\ \ y(0) = 200}$$

We can take advantage of the fact that y and y-70 have the same derivative, to rewrite this boxed equation as $\frac{d}{dt}(y - 70) = k (y - 70)$.

If so, $(y - 70) = C e^{kt}$, or $y = 70 + C e^{kt}$ where C is some constant.

Substitution by t = 0 , y = 200 gives C = 130.
Substitution by t = 10 , y = 150 gives $130 e^{10k} = 150 - 70$, or k = - .04855.

Thus $y = 70 + 130 e^{-.04855 t}$ and so when t = 27 , y = 105.04° .

ANSWER: At 1:27 p.m. the coffee temperature is 105.04° F