Math 105 Week 6 Learning Goals

1 Overview

We introduce integration by parts as a companion to the substition rule. These are the two most important integration techniques to evaluate definite integrals. For instance, they can be applied to prove that the circle of radius $r$ has area $\pi r^2$.

Topics to be covered include:

• Integration by parts for indefinite integrals: statement and understanding as the “reverse” of the product rule (7.2, p. 516)

• Integration by parts computation: simple examples (e.g. Examples 1-2)

• Integration by parts computation: repeated use to end up at a simpler integral (e.g. Example 3) or to solve for your original integral (e.g. Example 4)

• Integration by parts computation: one part trivial (e.g. $\int \ln x \, dx$, as in Example 5, or the antiderivative of inverse trigonometric functions)

• Integration by parts for definite integrals

Remark: we will not cover solids of revolution (Example 6, pp. 519-520) or reduction formulas (problems 44-47, p. 521)

2 Learning Objectives

By the end of the week, having participated in lectures, worked through the indicated sections of the textbook and other resources, and done the suggested problems, you should be able to:

1. state the integration by parts rule for (in)definite integrals and apply the integration by parts rule to transform a given (in)definite integral into a more simple (in)definite integral. [Procedural]

2. understand that sometimes integration by parts needs to be applied more than once, and how to handle these problems. [Procedural]

3. be able to choose between substitution rule and integration by parts when evaluating an integral. This isn’t always easy, but students should understand common formats of integrals that yield to each method.

Example problem: Evaluate the following integrals, using either the substitution method or integration by parts.

\[
(a) \int \frac{x}{\sqrt{x+1}} \, dx \quad (b) \int x^3(x^4 - 3)^6 \, dx \quad (c) \int \sin^2 \theta \, d\theta
\]
(d) \int e^x \cos(x) \, dx \quad (e) \int x^2 \ln^2 x \, dx \quad (f) \int \ln x \, dx