1 Overview

Last week we started on infinite series. In particular we learned about two
tests of convergence and/or divergence: namely the divergence test and the
integral test. This week we will learn a few more such tests designed to verify
convergence or divergence of an infinite series, even when we cannot evaluate the
series exactly. We will then begin the study of a very special class of functions
whose functional forms are given by infinite series.

- Ratio test, comparison test, and limit comparison test (8.5, pp. 641-647)
  (If students haven’t already been introduced to factorials, now is the time.)
  Remark: we won’t cover the root test

- Absolute and conditional convergence: definition, and theorem that abso-
lute convergence implies convergence (Theorem 8.21, p. 654).

- Using Theorem 8.21 to apply tests that only work with positive terms to
  series with mixed-sign terms. (e.g. Example 3(c), p. 655)
  Remark: we will not cover alternating series.

- Informal definition of a power series and the centre of a power series (9.1,
p. 661)

- Review: Taylor Polynomials (9.1, pp. 664-668)

- Taylor’s Remainder Theorem (9.1, p. 668) and estimating the remainder
  (9.1, p. 669)

- Definition of a power series and associated coefficients, centre, interval of
  convergence, and radius of convergence (9.2, p. 676)

- Theorem 9.3, Convergence of Power Series (9.2, p. 678); we won’t concern
  ourselves with the endpoints of the interval of convergence, but students
  should be able to find the maximum open interval over which the power
  series converges.

- Combining power series, Theorem 9.4 (9.2, p. 679)

- Differentiation and integration of power series, Theorem 9.5 (9.2, p. 680)

2 Learning Objectives

These should be considered a minimum, rather than a comprehensive, set of
objectives. By the end of the week, having participated in lectures, worked
through the indicated sections of the textbook and other resources, and done
the suggested problems, you should be able to independently achieve all of the
objectives listed below.
1. state the ratio test, comparison test and limit comparison test for convergence or divergence of an infinite series with positive terms. [Recall/Conceptual]

2. use the tests learned so far to check if an infinite series converges or diverges. Be able to recognize forms that indicate the applicability or otherwise of certain tests. [Procedural]

3. give the definition of absolute and conditional convergence. Give examples of some absolutely convergent and some conditionally convergent series (p.654). [Recall] Reading: Text §8.5 (pp. 641 – 647, excluding the portion on root test)

4. given a function $f$ that can be differentiated up to any order, describe the $n$th-order Taylor polynomial for $f$ with center $a$. [Procedural]

5. given a function $f$ that can be differentiated up to any order, explain why the $n$th order Taylor polynomial may be considered a good approximation to $f$. Use Taylor's theorem and Taylor's remainder formula to estimate the error in approximation of $f$. [Procedural/Conceptual]

6. define a power series, and the radius of convergence of a power series. Use the radius of convergence to determine an interval where the series converges absolutely. [Recall]

7. given a power series, be able to find the radius of convergence of a power series. [Procedural]

8. Be comfortable with algebraic manipulations of power series, such as sums, differences, multiplication by a power, composition, integration and differentiation. [Procedural]

Reading: Text § 9.2 (pp. 675 – 682)