

# Sample Midterm I for Math 104 and 184

[10] 1.

Compute the following limits:

a) (2 marks)  $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 8x + 15}$

b) (3 marks)  $\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{2x - 1} - 1}$

Compute the derivatives of the following functions.

c) (2 marks) Find  $f'(x)$  where  $f(x) = (x^2 + 7x)(e^x + x^3 + 2x^2 + 1)$ .  
DO NOT SIMPLIFY YOUR ANSWER.

d) (3 marks) Find  $h'(1)$  where  $h(x) = \frac{xf(x) - 5}{g(x)}$ ,  $f'(1) = 2$ ,  $g'(1) = -3$  and  $f(1) = 1$ ,  
 $g(1) = 1$ . EXPRESS YOUR ANSWER AS AN INTEGER.

[7] 2. Let  $f(x)$  be a function defined for all  $x$  near some number  $a$ .

(a) (2 marks) Carefully state the definition of  $f'(a)$ , the derivative of  $f(x)$  at the point  $x = a$ .

(b) (5 marks) Suppose  $f(x) = \frac{2}{x-1}$ . Show that  $f'(a) = \frac{-2}{(a-1)^2}$  using the definition of the derivative. NO credit will be given for any other method.

[8] 3. A spaceship travels along a path given by the graph  $y = x^2 \sin(x)$  in the plane.

a) (2 marks) What is the  $y$  coordinate of the ship when  $x = \pi$ ?

b) (6 marks) At the point in part (a), a piece breaks off of the ship and travels along the tangent line to the ship's path at this point. What is the  $y$  coordinate of this piece when its  $x$  coordinate is  $x = 2\pi$ ?

[8] 4. ABC Inc. has recently introduced the ABC smartphone. They anticipate that if they sell the smartphone at the price of \$300 per unit, they will sell 5000 units per week. For each \$10 increase in the price, they anticipate selling 200 fewer units per week. The fixed costs of producing the smartphone are \$100000 per week, and each smartphone costs ABC \$50 to make.

- a) (2 marks) Let  $p$  be price and  $q$  be weekly demand for the smartphone. Find the linear demand function  $p(q)$ .
- b) (1 marks) Find the weekly cost function  $C(q)$ .
- c) (2 marks) The weekly profit function  $P(q)$  is given by  $P(q) = -\frac{1}{20}q^2 + 500q - 100000$ . Find the marginal weekly profit function  $MP(q)$  (The Marginal Profit function is just the derivative of the profit function:  $MP(q) = P'(q)$ ).
- d) Suppose that the price is currently \$200 per unit. If the price is increased by a small amount,
- i) (1 marks) Will the quantity demanded increase or decrease? (explain)
- ii) (2 marks) Will the weekly profit increase or decrease? (explain)

[8] 5. Consider the function

$$f(x) = \begin{cases} ax^2 + 1 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ bx^3 + cx & \text{if } x > 1. \end{cases}$$

- (a) (2 marks) Compute the left and right hand limits of  $f(x)$  as  $x \rightarrow 1$ .
- (b) (2 marks) What equations must  $a, b$  and  $c$  satisfy so that  $f$  is continuous at  $x = 1$ ?
- (c) (4 marks) What equations must  $a, b$  and  $c$  satisfy for  $f$  to be differentiable at  $x = 1$ ? Determine the values of  $a, b, c$  for which  $f$  will be differentiable at  $x = 1$ .

[9] 6. For each question below, either explain why the statement is true or show the statement is false by providing a counter example if appropriate. No credit will be given for answers without justification.

(a) The equation  $\frac{x^3 - 2\sqrt{x} - 5}{e^x} = 0$  has a solution.

(b) The following function is continuous at  $x = 3$

$$f(x) = \begin{cases} \frac{x^2 + 1}{x - 1}, & \text{if } x \neq 3, \\ 5, & \text{if } x = 3, \end{cases}$$

(c) If  $f(x) + g(x)$  is differentiable at  $x = 1$ , then both  $f(x)$  and  $g(x)$  must also be differentiable at  $x = 1$ .