1. (a) \(3(x-3) + 1 \cdot (y-4) - 6(z+2) = 0\)
   
   \(\text{or } 3x+y-6z = 25\).

   (b) \(\vec{w} = <1, -4, 0>\) because
   
   \(\vec{u} \cdot \vec{w} = <4, 1, -3> \cdot <1, -4, 0> = 4 - 4 + 0 = 0\).

   (Students can use any of the following vectors:
   
   \(<a, 3b-4a, b>\), \(a, b\) are constants and \(ab \neq 0\))

2. (a) Since \((3, a)\) is in the domain of \(f(x, y)\), we have

   \[9 - x^2 - a^2 \geq 0 \Rightarrow -a^2 \geq 0 \Rightarrow a = 0.\]

   (b) \(f(x, y) = z_0 \Rightarrow \frac{1}{1 + \sqrt{9-x^2-y^2}} = z_0 \Rightarrow 1 + \sqrt{9-x^2-y^2} = \frac{1}{z_0}\)

   \(\Rightarrow \sqrt{9-x^2-y^2} = \frac{1}{z_0} - 1 \Rightarrow 9-x^2-y^2 = \left(\frac{1}{z_0} - 1\right)^2\)

   \(\text{or } x^2+y^2 = 9 - \left(\frac{1}{z_0} - 1\right)^2. \quad (1)\)

   If \(z_0 = \frac{1}{3}\), then \((1)\) becomes \(x^2+y^2 = 9 -(3-1)^2 = 5\), which is a circle.

   If \(z_0 = \frac{1}{4}\), then \((1)\) becomes \(x^2+y^2 = 9 -(4-1)^2 = 0 \Rightarrow (x, y) = (0, 0)\).
3. (a) \[ f_x = 10x \ln(5x^2y + 4y^2) + \left(5x^2 + 4y\right) \frac{1}{5x^2y + 4y^2} (10xy) \]

\[ = 10x \ln(5x^2y + 4y^2) + \left(5x^2 + 4y\right) \frac{1}{(5x^2 + 4y)y} (10xy) \]

or \[ f_x = 10x \ln(5x^2y + 4y^2) + 10x \]

(b) \[ f_{xy} = \frac{\partial}{\partial y} (f_x) = \frac{\partial}{\partial y} \left(10x \ln(5x^2y + 4y^2) + 10x\right) \]

\[ = 10x \cdot \frac{5x^2 + 8y}{5x^2y + 4y^2} \]
4. (a) \[ f_x = ye^{xy}, \quad f_y = ye^{xy} \]
\[ f_y = e^{xy} + ye^{xy}, \quad x = (1 + xy) e^{xy}. \]

The system

\[ \int f_x = ye^{xy} = 0 \quad (1) \]
\[ f_y = (1 + xy) e^{xy} = 0 \quad (2) \]

does not have a solution. In fact, (1) implies

\[ y^2 = 0 \quad \text{or} \quad y = 0 \quad (3) \]

and (2) implies that \( 1 + xy = 0 \) \( (4) \).

Using (3) and (4), we get \( 1 + x \cdot 0 = 0 \), which is
impossible. Hence \( f(x, y) \) does not have any critical point.

(b) \[ f_x = 6x, \quad f_y = A + 10y \]

Since \( \int f_x(0, 2) = 0 \)
\[ f_y(0, 2) = 0, \quad \text{we get} \quad A + 10(2) = 0 \Rightarrow A = -20. \]

Also, \( 3 = f(0, 2) = -20(2) + 5(2) + B = -40 + 10 + B \)
\[ \Rightarrow B = 23. \]
5. (a) We need to solve the system
\[
\begin{align*}
    f_x &= 3x^2 + 6y - 9 = 0 \\
    f_y &= 6x - 6y = 0
\end{align*}
\] (1) (2)

By (2), \( y = -x \) (3)

Using (3), (1) becomes \( 3x^2 + 6(-x) - 9 = 0 \) \( \Rightarrow x = 1 \) or \( x = -3 \).

Hence, all critical points are \( (1, -1) \) and \( (-3, 3) \).

(b) \( f_{xx} = 6x, \ f_{yy} = -6, \ f_{xy} = -6 \).

<table>
<thead>
<tr>
<th></th>
<th>( f_{xx} )</th>
<th>( f_{yy} )</th>
<th>( f_{xy} )</th>
<th>( D = f_{xx} f_{yy} - f_{xy}^2 )</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, -1)</td>
<td>6</td>
<td>-6</td>
<td>-6</td>
<td>-72</td>
<td>Saddle point</td>
</tr>
<tr>
<td>(-3, 3)</td>
<td>-18</td>
<td>-6</td>
<td>-6</td>
<td>72</td>
<td>Local maximum</td>
</tr>
</tbody>
</table>
6. (a) Objective function is \( Z = f(x, y) = x \)
Constraint is \( g(x, y) = x^2 + 6y^2 + 3xy - 40 = 0 \)
( or \( g(x, y) = x^2 + 6y^2 + 3xy = 40 \) )

We need to solve the system \( \begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ g(x, y) = 0 \end{cases} \)

\( f_x = x, \quad f_y = 6y + \lambda x \)

By (1) and (2), \( f_y = \lambda x \neq 0 \). Hence, (2) becomes

\( 12y + 3x = 0 \implies x = -4y \) \( (4) \)

Using (4), (3) becomes \((-4y)^2 + 6y^2 + 3(-4y) y = 40 \implies y^2 = 4 \implies y = \pm 2 \).

\( y = 2 \implies x = -4y = -8 \); \( y = -2 \implies x = -4(-2) = 8 \).

Since \( f(-8, 2) = -8 \) and \( f(8, -2) = 8 \), we get that the maximum value of \( f(x, y) = x \) is 8, and the minimum value of \( f(x, y) = x \) is -8.

(b) By (a), the point is \((8, 2)\).