# Solutions

1. (a) $\lim_{x\to -3} \frac{2x^2+5x-3}{x^2+x-6} = \lim_{x\to -3} \frac{(2x-1)(x+3)}{(x-2)(x+3)}$

$= \lim_{x\to -3} \frac{2x-1}{x-2} = \lim_{x\to -3} \frac{2(-3)-1}{-3-2} = \boxed{\frac{7}{5}}$.

(b) $\lim_{q\to\infty} R = \lim_{q\to\infty} (pq) = \lim_{q\to\infty} \frac{3400q^2+q}{2q^2+q} =$

$= \lim_{q\to\infty} \frac{3400q^2}{2q^2} = \boxed{1700}$

(c) $3000 = Pe^{(0.08)(2.5)}$, where $P$ is the initial amount. Hence, $P = \boxed{3000e^{-(0.08)(2.5)}}$.

(d) $y' = f'(g(x^2))\, g'(x^2)\, 2x$. Hence, $y'(1) =$

$= f'(g(1))\, g'(1)\cdot 2 = f'(0)(-1)\cdot 2 = 3(-1)\cdot 2 = \boxed{-6}$.

(e) $y' = \frac{3}{1+(3x)^2}$. We need to find $x$ such that

$\frac{3}{2} = y'(x) = \frac{3}{1+9x^2}$. Solving the equation,

we get $x = \boxed{\pm\frac{1}{3}}$.

(f) $\frac{dw}{dt} = 216\left(-\frac{1}{2}\right)(t+5)^{-3/2} + \frac{3}{2}t^{1/2}$. At $t=4$,

we have $\frac{dw}{dt} = -108\,(4+5)^{-3/2} + \frac{3}{2}\left(4^{1/2}\right) = -\frac{108}{27} + 3 =$

$= -4 + 3 < 0$. Hence, the animal is

$\boxed{\text{losing weight}}$.

(g) $R(520) - R(500) \approx R'(500)(520-500) = (2.5)(20) = \boxed{50}$.

1. (h) — (n).

(h). $\displaystyle\lim_{h\to 0}\frac{3h}{f(2)-f(2+h)} = \lim_{h\to 0}\frac{-3}{\frac{f(2+h)-f(2)}{h}}$

$\displaystyle = \frac{-3}{f'(2)} = \frac{-3}{6} = \boxed{-\frac{1}{2}}.$

(i) $\displaystyle f'' = \frac{2(1+x^2)-2x(2x)}{(1+x^2)^2}$

$\displaystyle = \frac{2(1-x^2)}{1+x^2}$ $\quad f''=0 \Rightarrow x=\pm 1.$

| | | | | | |
|---|---|---|---|---|---|
| $f''$ | $-$ | | $+$ | | $-$ |
| $f$ | | $-1$ | CU | $1$ | |

Hence, $f(x)$ is CU on $\boxed{(-1,1)}$.

(j) $f'(x) = 3x^2+2ax+b$. Hence, we have $0 = f'(0) = b$ and $0 = f'(1) = 3+2a+b$. Thus,

$$\boxed{a = -\frac{3}{2} \text{ and } b = 0}.$$

(k) Let s be the distance. Then

$$s = \sqrt{(x-6)^2 + y^2} = \sqrt{(x-6)^2 + x^2 - 4} = \sqrt{2x^2 - 12x + 32}.$$

$$\frac{ds}{dt} = \frac{4x - 12}{2\sqrt{2x^2 - 12x + 32}}. \qquad \frac{ds}{dt} = 0 \Rightarrow x = 3.$$

When $x = 3$, $3^2 - y^2 = 4$ or $y = \pm\sqrt{5}$. Thus,

$\boxed{(3, \sqrt{5})}$ is the point we need.

(l) $R = pq = 2000pe^{-kp}$. $\quad \dfrac{dR}{dp} = 2000e^{-kp} - 2000pke^{kp}$.

At $p = 200$, $0 = \dfrac{dR}{dp}$, which gives that

$$1 - 200k = 0 \text{ or } k = \boxed{\frac{1}{200}}.$$

(m) $f' = \dfrac{-\sin x}{2 + \cos x}$. $\qquad f'' = \dfrac{-\cos x(2 + \cos x) + \sin x(-\sin x)}{(2 + \cos x)^2}$

$$f''(0) = \frac{-(2+1) + 0}{(2+1)^2} = -\frac{1}{3}. \quad \text{Hence, } c_2 = \frac{f''(0)}{2!} = \boxed{-\frac{1}{6}}.$$

(n) The limit $= \lim_{x \to \infty} \dfrac{3x^5 - 2x + 7}{2x^5 - 3} \lim_{x \to \infty}\left(\dfrac{1}{e^x} + 5\right) = \left(\dfrac{3}{2}\right)(0 + 5) = \dfrac{15}{2}$

2, $\frac{df}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{3-(x+h)}} - \frac{1}{\sqrt{3-x}}}{h}$

$$= \lim_{h\to 0} \frac{\sqrt{3-x} - \sqrt{3-(x+h)}}{h\left(\sqrt{3-(x+h)}\right)\left(\sqrt{3-x}\right)}$$

$$= \lim_{h\to 0} \frac{(3-x) - (3-(x+h))}{h\sqrt{3-(x+h)}\sqrt{3-x}\left(\sqrt{3-x} + \sqrt{3-(x+h)}\right)}$$

$$= \lim_{h\to 0} \frac{h}{h\sqrt{3-(x+h)}\sqrt{3-x}\left(\sqrt{3-x} + \sqrt{3-(x+h)}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{3-(x+h)}\sqrt{3-x}\left(\sqrt{3-x} + \sqrt{3-(x+h)}\right)}$$

$$= \frac{1}{\sqrt{3-x}\sqrt{3-x}\left(\sqrt{3-x} + \sqrt{3-x}\right)} = \frac{1}{2(3-x)^{3/2}}.$$

#3.

(i) (4) $f'(x)=0 \Rightarrow x=0$ or $x=2$, which are the critical points of $f(x)$.

$f'$: $-$ | 0 | $+$ | 2 | $-$

$f$: $\searrow$ | 0 | $\nearrow$ | 2 | $\searrow$

Hence, $f(x)$ is decreasing on $(-\infty, 0)$ and $(2, \infty)$.

(ii) (5) $f''(x) = (2-2x)e^{-x} + (2x-x^2)e^{-x}(-1) = (x^2-4x+2)e^{-x}$.

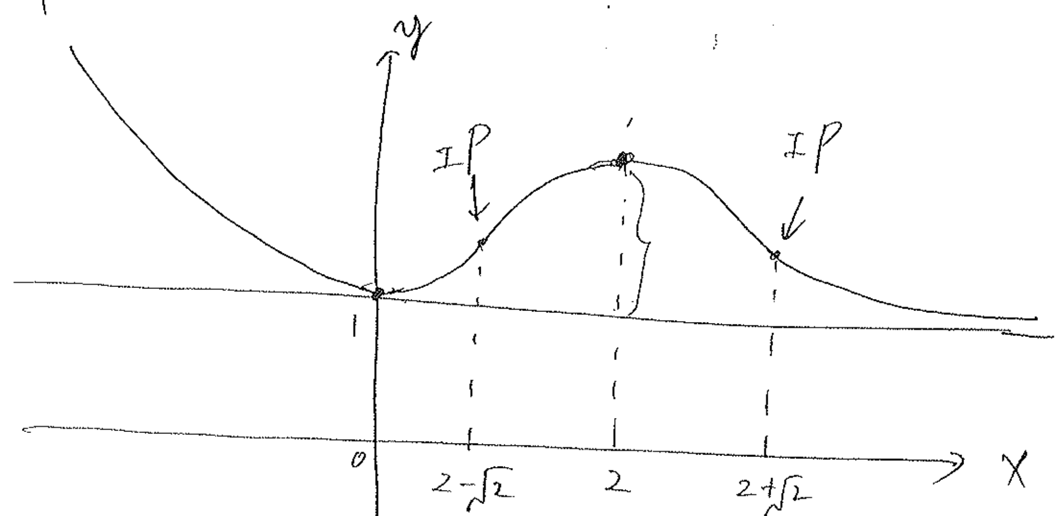
$f''=0 \Rightarrow x^2-4x+2=0 \Rightarrow x = \dfrac{4 \pm \sqrt{(-4)^2-4(2)}}{2} = 2 \pm \sqrt{2}$.

$f''$: $+$ | $2-\sqrt{2}$ | $-$ | $2+\sqrt{2}$ | $+$

$f$: CU | $2-\sqrt{2}$ | CD | $2+\sqrt{2}$ | CU

Hence, $f(x)$ is concave down on $(2-\sqrt{2}, 2+\sqrt{2})$.

(iii) (5)




$f(0)$ is a local minimum.

$f(2)$ is a local maximum

$(2 \pm \sqrt{2}, f(2 \pm \sqrt{2}))$ are IPs.

6

4. Let $r$ denote the radius of the top (or bottom), $h$ the height, and $C$ the cost. The goal is to minimize cost. We have

$$C = 3\pi r^2 + 3\pi r^2 + 2\cdot 2\pi r h = 6\pi r^2 + 4\pi r h.$$

Since $\pi r^2 h = 375\pi$, we have $h = \frac{375}{r^2}$. Hence,

$$C = 6\pi r^2 + 4\pi r \cdot \frac{375}{\cancel{\pi} r^2} = 6\pi r^2 + \frac{1500\pi}{r}$$

$$\frac{dC}{dr} = 12\pi r - \frac{1500\pi}{r^2} = \frac{12\pi(r^3 - 125)}{r^2}$$

$$\frac{dC}{dr} = 0 \Rightarrow r = \sqrt[3]{125} = 5.$$

Thus, $r = 5$ produces the minimum cost.

5. (i) Let $s$ be the distance between the moving point $(x, 0)$ and $(0, 1)$. Thus, $s = \sqrt{x^2+1}$.

Differentiating both sides with respect to time $t$, we get $\frac{ds}{dt} = \frac{x}{\sqrt{x^2+1}} \frac{dx}{dt} = \frac{5x}{\sqrt{x^2+1}}$.

If $\frac{ds}{dt} = 4$, then $4 = \frac{5x}{\sqrt{x^2+1}}$. Solving this equation, we get $x = \frac{4}{3}$.

(ii) (b) If $\frac{ds}{dt} = 6$, then $6 = \frac{5x}{\sqrt{x^2+1}}$, which does not have a solution. Thus, it does not exist a point at which the distance is increasing at a rate of 6 units per second.

6. Let $f(t)$ be the CPI in the country at time $t$

$$f(t) \approx f(0) + f'(0)t + \frac{f''(0)}{2}t^2$$

Hence, the CPI in the country 6 moths from now

$$= f(\tfrac{1}{2}) \approx f(0) + f'(0)\tfrac{1}{2} + \frac{f''(0)}{2}(\tfrac{1}{2})^2$$

$$= 120 + 10(\tfrac{1}{2}) + \frac{4}{2}(\tfrac{1}{2})^2 = 120 + 5 + 0.5 = 125.5$$