## MATH 104 & MATH 184: Week 0 Learning Goals

In this introductory week, we will review some material on exponential functions, logarithms, and inverse functions. This material is found in Chapter 1.3.

The natural exponential and logarithm functions as these are extremely important in this course. Note that we will revisit these functions in more detail later in the course, but it is good to have early exposure and multiple hits at this material.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. explain what an exponential function is. You should know the basic properties of exponential functions as presented in section 1.3. You should be able to graph exponential functions. You should be able to solve basic equations, as per the exercises, involving exponential functions;

2. explain what a one-to-one function is and how to test for this property graphically using the Horizontal Line Test;

3. explain what an inverse function is. You should be able to determine the intervals on which a given function has an inverse (if they exist). Given the graph of function, you should be able to graph the inverse function, if it exists;

4. describe the logarithmic functions as inverse functions of the exponential functions. You should know the basic properties of logarithmic unctions presented in section 1.3 that parallel those of the exponential functions. You should be able to graph logarithmic functions. You should be able to solve basic equations using logarithmic functions.

## MATH 104 & MATH 184: Week 1 Learning Goals

This week we introduce some concepts from business: revenue, costs, and profit, and introduce the students to their first optimization problem. This problem motivates the need for the derivative. Before formally introducing the derivative (week 2), we cover the material of Chapter 2.1, 2.2, and part of 2.3 to get an intuitive introduction to limits.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. explain revenue, costs, and profit for the case of linear demand. You should be able to set up and solve a simple problem involving maximizing revenue, for example, in this context. Note there are some posted notes on this material on the main Math 104/184 website;

2. give an intuitive explanation of the process of taking a limit. You should be able to compute the limits of functions in simple cases, as presented in section 2.2 and the beginning of 2.3 (up to and including the limits of polynomials and rational functions  end of page 67);

3. compute the average rate of change of a function. You should be able to draw a diagram that illustrates this quantity;

4. draw a diagram to illustrate the process of computing an instantaneous rate of change of a function;

5. explain the relationship between finding average and instantaneous rates of change of a function and appropriate secant and tangent lines on graphs of this function;

6. explain one-sided limits and their relationship to two-sided limits. You should be able to examine these limits graphically and numerically.

**MATH 104 & MATH 184: Week 2 Learning Goals**

This week we introduce continuity and the derivative. This is the material in section 2.6, 3.1 and 3.2 of Briggs Cochran. Please note that we have skipped sections 2.4 (infinite limits) and 2.5 (limits at infinity) and will return to these section when we do curve sketching. Questions from the text which are related to a learning goal are included as [section: question #s]. In some cases, a single question uses skills from several learning goals. The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. explain what it means for a function to be continuous at a point. you should be able to correctly analyze whether a given function is continuous at a given point. [2.6: 18,23];

2. identify points of discontinuity for a given function. [2.6:10];

3. know the way continuous functions behave under basic algebraic operations, and use these results to correctly identify whether or not a given function is continuous at a point. (See Theorem 2.9.) [2.6: 41-56];

4. know the way continuous functions behave when they are composed. (See Theorem 2.11.) [2.6:41 - 56];

5. identify whether or not a given function is continuous on a given interval. This includes identifying when a function is left- or right-continuous at the endpoints of a closed interval. [2.6: 39, 65];

6. state the Intermediate Value Theorem and to apply it to simple situations such as determining whether or not a function has a zero in some interval. [2.6: 8, 58, 65];

7. compute the average rate of change of a function on an interval. [3.1: 2, 56];

8. explain the notion of instantaneous rate of change at a given point and its role as the slope of the tangent line at that point. [3.1: 1, 56];

9. state the definition of the derivative and use it to compute the derivative of a given function in simple cases (such as those given in the exercises). [3.1: 9- 14, 10, 23];

10. sketch the graph of $f'$ given the graph of $f$. [3.2: 9, 10];

11. explain using sketches of appropriate functions the relationship between continuity and dierentiability. [3.1: 63].

## MATH 104 & MATH 184: Week 3 Learning Goals

This week we introduce some of the basic rules of differentiation. This is the material in sections 3.3 and 3.4 of Briggs Cochran. We will also introduce derivatives of the basic trigonometric functions, but only as a table of derivatives (Theorem 3.13 in section 3.5).

Suggested problems that help build these skills are given as [section: question #s]. The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. use the power, sum, and constant multiple rules to differentiate, for example, polynomials. [3.3:19-24,28,33,54];

2. use the derivative of an exponential function. [3.3: 38,72,79; 3.4:54];

3. know the definition of "$e$" as the base of the exponential function with the property that $\frac{d}{dx}(e^x) = e^x$.[3.3:79];

4. correctly state and use the product rule. [3.4:7-14,15];

5. correctly state and use the quotient rule. [3.4:27,31,51,82];

6. differentiate given functions using appropriate combinations of the rules of differentiation. [3.4:59,60];

7. find equations of tangent lines to given functions at given points. [3.3:41; 3.4:34,72];

8. compute higher-order derivatives. [3.4:87];

9. compute derivatives involving the basic trigonometric functions. [3.5: 6,15 - 28, 46,62,66].

## MATH 104 & MATH 184: Week 4 Learning Goals

This is midterm week 4. This week we work with the derivative as rate of change and the chain rule. This is the material in sections 3.6 and 3.7 of Briggs Cochran. We will be interested in Growth Models and Average and Marginal Cost/Revenue/Profit/etc. Indeed, we will relate the idea of "marginal" from economics with the idea of derivative in this course. Note that there are some notes on this for you in the Notes on a Business Problem that are posted on the main course webpage.

Suggested problems that help build these skills are given as [section: question #s]. There are also some extra problems posted on the business related concepts.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. work with velocity and acceleration as derivatives. [3.6:10,11,12,35];

2. solve problems involving average and instantaneous growth rates. [3.6: 20,41];

3. explain the notion of marginal cost (or revenue or profit, etc.) in terms of the derivative of the cost (or revenue or profit) function. [3.6:7 plus posted extra problems];

4. solve problems involving average and marginal costs (revenue/profit/etc.). [3.6: 21,42, 46,47 plus posted extra problems];

5. state the Chain Rule, including its hypotheses, and identify when it can be used. [3.7:2-6] 6. make use of the Chain Rule in computations. [3.7: 35,36,38, 50,51,52,79, 81, 82, 88, 93, 99, 100].

## MATH 104 & MATH 184: Week 5 Learning Goals

We will cover implicit differentiation this week, section 3.8 of Briggs Cochran. We will cover up to and including the power rule for rational exponents on p. 199. We will also begin section 3.9 of Briggs Cochran, which is on the derivatives of logarithmic and exponential functions. The text approaches this through the inverse function relationships between logarithms and exponentials.

Suggested problems that help build these skills are given as [section: question #s].

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. explain what we mean byimplicit differentiation and identify situations where they will use it. [3.8: 2,3];

2. carry out computations involving implicit differentiation. [All assigned problems except 2 and 3.];

3. find equations of tangent lines to graphs of implicitly de
ned functions. [3.8: 27,28,54,56];

4. find equations of normal lines to graphs of implicitly defined functions. [3.7: 68,74];

5. use the implicit differentiation to demonstrate the power rule for rational exponents;

6. work with the inverse properties of $e^x$ and $\ln x$;

7. use the derivatives of general logarithmic functions in computations;

8. use the derivatives of general exponential functions in computations.

## MATH 104 & MATH 184: Week 6 Learning Goals

This week has three key topics. We will finish section 3.9 of Briggs Cochran (which we began last week), which is on the derivatives of logarithmic and exponential functions. We will cover *price elasticity of demand*, which is not covered in the text. While such elasticities can be a big topic, we will cover it in an introductory way. You find the material in the notes I have post online on price elasticity of demand. Finally, we will introduce exponential growth, especially as applied to continuous compound interest. This material is in section 6.8 and a short set of notes on continuous compound interest, which are in **Supplementary notes**.

The suggested problems will help you build these skills. I will post an extra set of problems for the price elasticity of demand section and for the continuous compound interest section.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. work with the inverse properties of $e^x$ and $\ln x$;

2. use the derivatives of general logarithmic functions in computations;

3. use the derivatives of general exponential functions in computations;

4. use the technique of logarithmic differentiation;

5. use the logarithmic derivative of a function $f(x)$ to compute relative rates of change of $f(x)$per unit change of $x$;

6. compute the *price elasticity of demand* and use it to determine the direction revenue changes when there is a change in price;

7. solve problems involving price elasticity of demand;

8. compare and contrast linear growth and exponential growth;

9. solve problems involving continuously compounded interest

## MATH 104 & MATH 184: Week 7 Learning Goals

This week we cover two sections of material. The first topic is Related Rates, which is in section 3.11 of Briggs Cochran. The second is Maxima and Minima, which is covered in section 4.1.

Suggested problems that help build these skills are given as [section: question #s].

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. set up and solve related rates problems; [3.11: 3, 10, 15, 19, 22, 24, 29.46]. By this, we mean: Given a draining tank, falling ladder or moving ship problem, or provided a model (including equation) of another situation, you should be able to:

(a) identify all the variables involved, make appropriate choices when a variable takes on con- stant values, and describe how they relate (using equations if relevant and/or writing ashort paragraph);

(b) draw a picture of the situation if needed;

(c) interpret rates in terms of derivatives with the appropriate variables; and

(d) derive an equation which describes how the relevant rates are related and solve in that equation for the desired target rate.

2. define absolute maximum and absolute minimum and give examples of functions that illustrate these concepts; [4.1: 1, 10]

3. state the Extreme Value Theorem, and give examples that illustrate their understanding of this theorem: (1) examples where the EVT applies, and (2) examples where the EVT does not apply, but functions have absolute maxima or minima; [4.1: 4,5,13,14]

4. define local maximum and local minimum and give examples of functions that illustrate these concepts; [4.1: 7,8,20]

5. define critical point and apply this definition to
nd and classify critical points of a given function; [4.1: 24,30,31,33,52,54,66]

6. find the absolute maximum and absolute minimum of a given continuous function on a closed interval. [4.1: 58,61,62]

**MATH 104 & MATH 184: Week 8 Learning Goals**

This week we cover the first and second derivative tests and curve sketching. This is material in sections 4.2 and 4.3 in Briggs Cochran, with some additional material from 2.4 and 2.5 focused on asymptotes.

Suggested problems that help build these skills are given as [section: question #s].

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. explain how the first derivative of a function determines where the function is increasing and decreasing and apply this to specific functions to determine their intervals of increase and decrease; [4.2: 1, 3, 12, 16, 22, 34, 83]

2. use the first derivative test to identify local maxima and minima; [4.2: 2, 40, 46, 47, 50]

3. explain how the second derivative of a function determines concavity and apply this to specific functions to determine where they are concave up and concave down, and to identify in ection points; [4.2: 60, 64,68, 70,100,83]

4. use the second derivative test to classify local maxima and minima; [4.2: 79,80, 97,98] 5. identify any asymptotic behaviours a function may have: vertical asymptotes, horizontal asymptotes, and oblique or slant asymptotes; [2.4: 9, 11, 21; 2.5: 28, 32, 52, 53, 57, 68]

6. use calculus to sketch a graph of a given function. [4.3: 2,3, 8, 13, 19, 25, 30, 35, 36, 48, 49, 70]

## MATH 104 & MATH 184: Week 9 Learning Goals

We will start to work with Optimization Problems in section 4.4 of Briggs Cochran in Week 9. There will be some extra business related optimization problems posted as well.

Some of you may be finishing up the learning goals from Week 8 in Week 9. This is fine as one week will be enough for Optimization Problems, if need be.

The specific learning goals for this section are that by the end of Week 10 and review homework, students should be able to:

1. interpret the idea of optimization as the procedure used to make a system or a design as effective or functional as possible, and translate it into a mathematical procedure for finding the maximum/minimum of a function;

2. set up an optimization problem by identifying the objective function and all appropriate constraints ; and

3. use calculus to solve optimization problems, and explain how they used the constraints in the solution process.

**MATH 104 & MATH 184: Week 10 Learning Goals**

We will finish our treatment on Optimization Problems in section 4.4 of Briggs Cochran. See the week 9 learning goals for suggested problems here. We will also start to look at linear approximations and differentials in this week in Section 4.5.

The specific learning goals for these sections are that by the end of Week 10 and review homework, students should be able to:

1. interpret the idea of optimization as the procedure used to make a system or a design as effective or functional as possible, and translate it into a mathematical procedure for finding the maximum/minimum of a function;

2. set up an optimization problem by identifying the objective function and all appropriate constraints; and

3. use calculus to solve optimization problems, and explain how they used the constraints in the solution process;

4. explain linear approximation (also known as tangent line approximation and the linearization of a function) using a series of figures like those in Section 4.5; this includes being able to relate the formula for linearization to the elements of such a picture (for example: what is the role of $a$, what is the role of $x$, where is the appropriate tangent line on the graph, where does the linear approximation appear in the picture); [4.5:2,3,4,47]

5. use linear approximations to estimate the values of functions near a given $x = a$; [4.5: 15, 16, 18, 24, 26, 30, 51, 57, 61, 63]

**MATH 104 & MATH 184: Week 11 and 12 Learning Goals**

Over the last two weeks of term, we will cover Approximation (Linear, Quadratic and the more gen- eral Taylor Polynomial), and finish with a discussion on inverse trigonometric functions. The Linear Approximation Material is in Section 4.5, and you may have already covered some of this last week. The Taylor Polynomial material will be from Section 9.1 of Briggs Cochran. The inverse trigonometric functions are covered in section 3.9 of Briggs Cochran. You should have covered the basics of inverse trigonometric functions in Math 12, and the main content of 3.10 for you will be the derivatives of $arcsin\, x$, $arccos\, x$, and $arctan\, x$.

Suggested problems that help build these skills are given as [section: question #s].

The specific learning goals for this week for Linear Approximation are that by the end of week 12 and review homework, you should be able to:

1. use linear approximations to estimate the values of functions near a given $x = a$; [4.5: 15, 16, 18, 24, 26, 30, 51, 57, 61, 63]

2. use linear approximation to approximate changes in the dependent variable given changes in the independent variable; [4.5:37, 38]

3. given the exact value, discuss the discrepancy with the linear approximation in terms of the second derivative (for example, whether it is an underestimate or overestimate); [4.5: Quick Check 1, 57]

4. analyze the worst-case error for a linear approximation of a function using a formula based on the second derivative of the function;

5. use the quadratic approximation to estimate the values of functions; [9.1: 9, 11]

6. explain the difference between the 2nd order term in quadratic approximation and the worst-case error term for linear approximation;

7. find the $n$th degree Taylor polynomial of a given function with a given centre $x = a$; [9.1: 17,21,31,73]

8. use a Taylor polynomial to approximate the values of functions; and [9.1: 39,42,43]

9. use the table of derivatives of inverse trigonometric functions in calculations of derivatives. [3.10:7 - 12, 22,26]