Solutions for Sec. 6.8: Exponential Models

6.8.1 Exponential growth occurs for a constant relative growth rate.

6.8.10 For $f(t)$, $\frac{df}{dt} = 400$, so the absolute growth rate is constant. For $g(t)$, $\frac{dg}{dt} = 400 \cdot 2^{t/20} \cdot \frac{\ln 2}{20}$, so the growth rate is not constant but the relative growth rate $\frac{1}{g(t)}g'(t) = \frac{\ln 2}{20}$ is constant.

6.8.11 The growth is modeled by $p(t) = 90,000e^{kt}$, with $t = 0$ corresponding to 2010, and time measured in years. Since $(1.024)(90,000) = 90,000e^k$, we have $k = \ln(1.024)$. The doubling time is $T_2 = \frac{\ln 2}{\ln(1.024)} \approx 29.2263$, so it should reach 180,000 around the year 2039.

6.8.13 The price of a cart of groceries is modeled by $p(t) = 100e^{kt}$ where $t = 0$ corresponds to 2005, and t is measured in years and $k = \ln(1.03)$. The price of a the groceries in 2015 when $t = 10$ should be about $p(10) = 100e^{10\ln(1.03)} \approx \$134.39$.

6.8.16

a. Note that $k = \ln(1.015)$. The amount consumed over the year would be

$$\int_0^1 1.2e^{\ln(1.015)t}\,dt = \left(\frac{1.2}{\ln(1.015)}e^{\ln(1.015)t}\right)\Bigg|_0^1 \approx 1.20898$$

million barrels of oil.

b. $\int_0^t 1.2e^{\ln(1.015)x}\,dx = \left(\frac{1.2}{\ln(1.015)}e^{\ln(1.015)x}\right)\Big|_0^t = \frac{1.2}{\ln(1.015)}(e^{\ln(1.015)t} - 1)$ million barrels.

c. This will occur when $\frac{1.2}{\ln(1.015)}(e^{\ln(1.015)t} - 1) = 10$, or $e^{\ln(1.015)t} = \frac{10\ln(1.015)}{1.2} + 1$, or

$$t = \ln\left(\frac{10\ln(1.015)}{1.2} + 1\right)/(\ln(1.015)) \approx 7.85551 \text{ years.}$$

6.8.25

a. False. If that was the correct formula, then $y(1) = y_0e^{.06} = (1.06184)y_0 \neq (1.06)y_0$.

b. False. If it increases by ten precent per year, then after 3 years it increases by a factor of $(1.1)^3 = 1.331$ which corresponds to 33.1 percent.

c. True. The relative decay rate is constant, so the decay is exponential.

d. True. This follows because the doubling time is related to k by the equation $T_2 = \frac{\ln 2}{k}$.

e. True. This time would be the constant $\frac{\ln 10}{k}$.

6.8.30 $T_2 = \frac{\ln 2}{k} = \frac{\ln 2}{\ln(1+.0p)} \approx \frac{.693}{\ln(1+.0p)} \approx \frac{70}{p}$. The approximation is best when $\ln(1 + .0p) \approx .0p$, which occurs when p is small.

6.8.38 If $y(p) = \sqrt{y(m)y(n)}$, then $y_0e^{kp} = \sqrt{y_0^2e^{mk}e^{nk}}$, so $e^{kp} = e^{(m+n)k/2}$, so $p = \frac{m+n}{2}$.