4.2. Let \( a, b \in \mathbb{Z} \), where \( a \neq 0 \) and \( b \neq 0 \). Prove that if \( a|b \) and \( b|a \) then \( a = b \) or \( a = -b \).

4.10. Let \( n \in \mathbb{Z} \). Prove that \( 2|(n^4 - 3) \) if and only if \( 4|(n^2 + 3) \).

4.18. Let \( m, n \in \mathbb{N} \) such that \( m \geq 2 \) and \( m|n \). Prove that if \( a \) and \( b \) are integers such that \( a \equiv b \pmod{n} \), then \( a \equiv b \pmod{m} \).

4.22. Let \( n \in \mathbb{Z} \). Prove each of the statements (a) – (f).

(a) If \( n \equiv 0 \pmod{7} \), then \( n^2 \equiv 0 \pmod{7} \).

(b) If \( n \equiv 1 \pmod{7} \), then \( n^2 \equiv 1 \pmod{7} \).

(c) If \( n \equiv 2 \pmod{7} \), then \( n^2 \equiv 4 \pmod{7} \).

(d) If \( n \equiv 3 \pmod{7} \), then \( n^2 \equiv 2 \pmod{7} \).

(e) For each integer \( n \), \( n^2 \equiv (7 - n)^2 \pmod{7} \).

(f) For every integer \( n \), \( n^2 \) is congruent to exactly one of 0, 1, 2 or 4 modulo 7.

4.28. Prove that if \( r \) is a real number such that \( 0 < r < 1 \), then \( \frac{1}{r(1-r)} \geq 4 \).

4.32. (a) Recall that \( \sqrt{r} > 0 \) for every positive real number \( r \). Prove that if \( a \) and \( b \) are positive real numbers, then \( 0 < \sqrt{ab} \leq (a + b)/2 \). (The number \( \sqrt{ab} \) is called the geometric mean of \( a \) and \( b \), while \( (a + b)/2 \) is called the arithmetic mean or average of \( a \) and \( b \).)

(b) Under what conditions does \( \sqrt{ab} = (a + b)/2 \) for positive real numbers \( a \) and \( b \)? Justify your answer.

4.34. Prove for every three real numbers \( x, y \) and \( z \) that \( |x - z| \leq |x - y| + |y - z| \).

4.38. Let \( a, b, x, y \in \mathbb{R} \) and \( r \in \mathbb{R}^+ \). Prove that if \( |x - a| < r/2 \) and \( |y - b| < r/2 \), then \( |(x + y) - (a + b)| < r \).

4.42. Let \( A \) and \( B \) be sets. Prove that \( A \cap B = A \) if and only if \( A \subseteq B \).

4.48. Let \( A = \{n \in \mathbb{Z} : 2|n\} \) and \( B = \{n \in \mathbb{Z} : 4|n\} \). Let \( n \in \mathbb{Z} \). Prove that \( n \in A - B \) if and only if \( n = 2k \) for some odd integer \( k \).

4.56. Let \( A, B \) and \( C \) be sets. Prove that \( (A - B) \cup (A - C) = A - (B \cap C) \).

Question. Let \( A, B \) be two sets. Prove: If \( A \cap B = \emptyset \), then \( A = (A \cup B) - B \).

4.58. Let \( A, B \) and \( C \) be sets. Prove that \( A \cap (B \cap C) = (A \cup B) \cap (A \cap C) \).

4.64. For sets \( A \) and \( B \), find a necessary and sufficient condition for \( (A \times B) \cap (B \times A) = \emptyset \). Verify that this condition is necessary and sufficient.

4.68. Let \( A, B, C \) and \( D \) be sets. Prove that \( (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D) \).