This midterm has 4 questions on 5 pages

- Read all the questions carefully before starting to work.
- Give complete arguments and explanations for all your calculations; answers without justifications will not be marked.
- Continue on the back of the previous page if you run out of space.
- Attempt to answer all questions for partial credit.
- This is a closed-book examination. None of the following are allowed: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

Full Name (including all middle names): 

Student-No: 

Signature: 
1. (a) Write the negation of the following statement:
   “There is some \((a, b) \in \mathbb{N} \times \mathbb{N}\) so that \(a \leq b\) and \(a^2 \leq b^2\).”

(b) Write the negation of the following statement:
   “The integer \(n\) is even if and only if \(n^2 + 1\) is even.”

(c) Write the converse and contrapositive of the following statement:
   “If \(n\) is even, then \(n + 1\) is odd.”

(d) Give a precise definition of a \(set\ partition\).
2. Determine whether the following six statements are true or false — explain your answers ("true" or “false” is not sufficient).

(i) \[ \forall x \in S, \forall y \in S, xy = 3. \]
(ii) \[ \forall x \in S, \exists y \in S \text{ s.t. } xy = 3. \]
(iii) \[ \exists x \in S \text{ s.t. } \forall y \in S, xy = 3. \]
(iv) \[ \exists x \in S \text{ s.t. } \exists y \in S \text{ s.t. } xy = 3. \]
(v) \[ \exists x \in S \text{ s.t. } \forall y \in S, \forall z \in S, \text{ if } z > y, \text{ then } z \geq x + y. \]
(vi) \[ \forall x \in S, \exists y \in S \text{ s.t. } \forall z \in S, \text{ if } z > y, \text{ then } z \geq x + y. \]
3. (a) Let $n \in \mathbb{Z}$. Prove that $7n + 1$ and $3n - 6$ have opposite parity.
   (That is, they cannot both be odd and they cannot both be even).
   
   (b) Prove that $a^3 + 2a \equiv 0 \pmod{3}$. 
4. Let $A, B, C$ be sets.
   
   (a) Prove that $A \cap B \subseteq A \cup B$.
   
   (b) Prove that $(A - B) \cup (A - C) \subseteq (A - (B \cap C))$.
   
   (c) Prove that $(A \cap B) \cup (A \cap C) \subseteq (A \cap (B \cup C))$. 