Problem 1: [6 points]

(a) Draw the polar curve \( r = 5 \sin(3\theta) \)

(b) Find the area of the region bounded by this curve in the first quadrant (that is \( 0 \leq \theta \leq \pi/3 \) for the curve).

\[
A = \int_0^{\pi/3} \frac{r^2}{2} \, d\theta = \int_0^{\pi/3} \frac{25 \sin^2 3\theta}{2} \, d\theta
\]

\[
= \frac{25}{2} \left[ \frac{1 - \cos 6\theta}{2} \right]_0^{\pi/3} = \frac{25}{4} \left[ \theta - \frac{\sin 6\theta}{6} \right]_0^{\pi/3}
\]

\[
= \frac{25 \pi}{12}
\]

(b) Find the the length of the whole curve when we trace it once (that is \( 0 \leq \theta \leq \pi \)). You can leave your solution as a definite integral.

\[
L = \int_0^\pi \sqrt{r^2 + r'^2} \, d\theta
\]

\[
= \int_0^\pi \sqrt{25 \sin^2 3\theta + 225 \cos^2 3\theta} \, d\theta
\]
Problem 2: [3 points] Find all solutions of \( y' = \frac{xe^x}{y} \).

This is a separable differential equation.

\[
\frac{dy}{dx} = \frac{xe^x}{y} \quad y \frac{dy}{dx} = xe^x \quad \int y \, dy = \int xe^x \, dx \\
y^2 = 2xe^x - 2e^x + c \quad y = \pm \sqrt{2xe^x - 2e^x + c}
\]

Problem 3: [3 points] A population of bacteria grows based on the following equation:

\[
\frac{dP(t)}{dt} = 5P(t)(3 - P(t))
\]

Assuming the initial population to be \( P(0) = 10 \), Find the population at time \( t \).

This is a separable differential equation.

\[
\frac{dP}{P(3-P)} = 5 \, dt \quad \int \frac{dP}{P(3-P)} = \int 5 \, dt
\]

\[
\int \left( \frac{\sqrt{3}}{P} + \frac{\sqrt{3}}{3-P} \right) \, dP = \int 5 \, dt
\]

\[
\frac{1}{3} \ln |P| - \frac{1}{3} \ln |3-P| = 5t + c
\]

\[
\ln \left| \frac{P}{3-P} \right| = 15t + c \quad \left| \frac{P}{3-P} \right| = e^{15t+c} = e^c e^{15t} = ce^{15t} \quad (\text{new c})
\]

\[
P = ce^{15t} (\text{nonzero const.}) \quad P = 3ce^{15t} - ce^{15t}
\]

\[
\frac{P}{3-P} = ce^{15t}
\]

\[
p(0) = 10 \implies c = -\frac{10}{7}
\]

\[
p(t) = \frac{30e^{15t}}{7-10e^{15t}}
\]