Problem 1: In each case, solve for $y(t)$:
(a) $y' + 3y = 2e^{t/2}$ with $y(0) = 1$.
(b) $y' - 4y = t$ with $y(1) = 0$.
(c) $ty' + 2y = \sin t$ with $y(\pi/2) = 0$.
(d) $t^2y' + 2ty = t^3 + 1$ with $y(1) = 1$.

Problem 2: Draw the direction fields for each of the below and sketch the solution curve corresponding to the indicated initial condition.
(a) $y' = y - 2\cos(t) + 1$, $y(0) = 0$
(b) $y' = \sqrt{y} - ty$, $y(0) = 2$

Problem 3: The following equation, called the logistic equation, is often used to describe the growth of a population $N(t) \geq 0$ with limited resources:
\[ \frac{dN}{dt} = rN\frac{(K - N)}{K} \]
Here $r, K > 0$ are constants (called the intrinsic growth rate and the carrying capacity, respectively).
(a) Define a new variable, $y(t) = N(t)/K$ and rewrite the logistic equation in terms of this new variable.
(b) Sketch a direction field for $t \geq 0, y \geq 0$ and include on it the solution curves for initial condition $y(0) = 0, y(0) = 0.2, y(0) = 0.8, y(0) = 1.6$. What happens as $t \to \infty$? Interpret in terms of the growing population (i.e. in terms of the original variable $N(t)$.)
(c) Find a mathematical argument that supports the following statement: “Only solutions curves with $y(0) \leq 1/2$ have an inflection point.”

Problem 4: According to Torricelli’s Law, the height of fluid in a container above a hole (through which the fluid is escaping) is governed by a differential equation:
\[ \frac{dh}{dt} = -k\sqrt{h}. \]
where \( k \geq 0 \) is a constant. Suppose the height of the fluid is initially \( h(0) = h_0 \). How long does it take for the fluid to drain to the level of the hole?

**Problem 5:** Set up the following two problems as initial value problems (i.e. differential equation and initial condition) and solve each one.

(a) In the LR circuit shown in Fig 1(a), \( R, L, V \geq 0 \) are constant. Find the current \( i(t) \) given that \( i(0) = 0 \).

(b) In the RC circuit shown in Fig 1(b), \( R, C \geq 0 \) are constant and the voltage is time dependent, \( V(t) = e^{-t} \). Find the charge on the capacitor \( q(t) \) given that \( q(0) = 0 \).

![Figure 1: For problem 5 (a) and 5 (b)](image)

**Problem 6:** Consider a stirred tank reactor that initially contains a volume \( V(0) = V_0 \) of water. Now suppose that a stock solution of salt (at concentration \( S \) gm/Litre) is pumped in at rate \( F_{in} = F \) Litres/hr and the well-stirred mixture is pumped out at a slightly faster rate, \( F_{out} = (F + f) \) Litres/hr where \( f > 0 \). [Note that the volume of the fluid will not be constant.] Let \( C(t) \) denote the concentration of salt inside the tank.

(a) Set up this problem as a differential equation problem for the volume of fluid \( V(t) \) and the salt concentration \( C(t) \).

(b) Determine the volume of fluid \( V(t) \) for \( t > 0 \). Over what period of time \( 0 \leq t \leq T \) is this result valid?

(c) Use your result in (b) to set up one ODE that depends only on the variable \( C(t) \) and solve that equation.