Laplace Transforms

What if initial conditions are not at time $t=0$?

Example: $w''(t) - 2w'(t) + 5w(t) = -8e^{-t}$

$w(0) = 2 \quad w'(0) = 12$

Need to first transform the problem, e.g. define $y(t) = w(t + \pi)$

Then $y'(t) = w'(t + \pi)$
$y''(t) = w''(t + \pi)$

$y(0) = w(\pi) = 2$
$y'(0) = w'(\pi) = 12$

Then replacing $t \rightarrow t + \pi$ in ODE: $w''(t + \pi) - 2w'(t + \pi) + 5w(t + \pi) = -8e^{-(t + \pi)} = -8e^{-t}$

on in terms of $y$:

\[
\begin{cases}
    y''(t) - 2y'(t) + 5y(t) = -8e^{-t} \\
y(0) = 2 \quad y'(0) = 12
\end{cases}
\]

We can now apply the usual Laplace Transform method (left as an exercise). We find that

$y(t) = 3e^{t}\cos(2t) + 4e^{t}\sin(2t) - e^{-t}$

To then find $w(t)$, use $w(t + \pi) = y(t) \Rightarrow y(t + \pi) = w(t)$

So

$w(t) = y(t) \bigg|_{t \rightarrow t - \pi} = 3e^{t - \pi}\cos(2(t - \pi)) + 4e^{t - \pi}\sin(2(t - \pi)) - e^{-(t - \pi)}$

* Thanks to Jessica Conway for this nice example