In-class problem:

\[ y'' + 4y = 5 \cos(3t) \quad y(0) = 0, y'(0) = 0 \]

- Comp. hom. prob. \[ y'' + 4y = 0 \]
  char. eqn \[ r^2 + 4 = 0 \]
  roots \[ r = \pm 2i \quad r^2 = -4 \]
  soln to hom prob.
  \[ y(t) = C_1 \cos(2t) + C_2 \sin(2t) \quad \text{"natural frequency"} \]

- Particular soln:
  guess \[ y_p(t) = A \cos(3t) + B \sin(3t) \]
  form of forcing function may need this term too
  This does not duplicate the solns to hom problem (since frequencies not same)
  So guess should be fine.

- Find A, B:
  need \[ y_p'(t) = -3A \sin(3t) + 3B \cos(3t) \]
  \[ y_p''(t) = -9A \cos(3t) - 9B \sin(3t) \]

  plus into \[ y'' + 4y = 5 \cos(3t) : \]
  \[ (-9A \cos(3t) - 9B \sin(3t)) + 4(A \cos(3t) + B \sin(3t)) = 5 \cos(3t) \]

  sort the terms:
  \[ \cos(3t) \left[ -9A + 4A \right] = 5 \cos(3t) \quad \Rightarrow -5A = 5 \quad A = -1 \]
  \[ \sin(3t) \left[ -9B + 4B \right] = 0 \quad \Rightarrow B = 0 \]

  so \[ y_p(t) = -1 \cdot \cos(3t) + 0 \cdot \sin(3t) = - \cos(3t) \]

- Genl soln:
  \[ y(t) = C_1 \cos(2t) + C_2 \sin(2t) - \cos(3t) \]
  soln to hom prob. \( \overline{\text{part. soln}} \)

- Use initial cond's
  \[ y(0) = 0 \quad \Rightarrow \quad \text{setup} C_1 - 1 = 0 \quad C_1 = 1 \]
  \[ y'(0) = 0 \quad \Rightarrow \quad C_2 = 0 \]

- Soln:
  \[ y(t) = \cos(2t) - \cos(3t) \]