Example 3  Direction Field for the ODE

\[ \frac{dy}{dt} = t - \sin y \quad y(0) = -1 \]

This quantity is the slope of a tangent line to solution curves (since it has to match the first derivative at any point \( y(t) \)).

To sketch this direction field, it is helpful to first sketch out \((t,y)\)-curves for which

\[ t - \sin y = M = \text{constant} \]

then this constant will tell us what are slopes of tangents to \( y(t) \) all along that locus.

Here are a few such curves (they are all sine curves shifted along the \( t \) axis's)

Next, let us draw in the tangent lines having slope \( M \) on each of these loci. This produces a direction field

Finally, let us sketch in a few solution curves \( y(t) \), i.e. curves that start at some initial condition and are always parallel to this "flow"

Note: Of course, we could just create a table of values for \( t \) \( y \) \( \frac{dy}{dt} \)

as in Example 2' in the lecture, but that gets quite tedious!

The above uses our graphical skills more efficiently.
Example 4: Draw a direction field for
\[
\frac{dy}{dt} = \frac{1}{2} y t \quad y(0) = 2
\]
This says that slope of tangent lines to solutions are
\[
M = \frac{1}{2} y t \quad \text{i.e.} \quad y = \frac{2M}{t}
\]
Let us sketch a few loci for which this relationship holds, where \( M \) is some constant.

Now add tangent vectors of slope \( M \) to each curve (with \( M = 1, 2, 3, -1, -2 \) etc).

We get the picture shown below.

Finally, add solution curves.

The curve we want satisfies \( y(0) = 2 \) and is shown in red.

See also Boyce + DiPrima (9th ed.) Sec 1.1