

Math102 Section 110

Instructor: L. Keshet

Your Name: \_\_\_\_\_

Worksheet for Oct 26, 2017

1. **Simple related Rates:** The volume and surface area of a sphere are  $V = \frac{4}{3}\pi r^3$  and  $S = 4\pi r^2$ .
  - (a) If the radius of the sphere is increasing at a constant rate, at what rate is the volume increasing?
  - (b) If the volume is increasing at a constant rate, is the radius also increasing at a constant rate? (Explain, by actually computing  $dr/dt$ .)

2. Use implicit differentiation to find the slope of the tangent line to the circle  $x^2 + y^2 = 1$  at the point  $x = 1/2$  in the first quadrant.

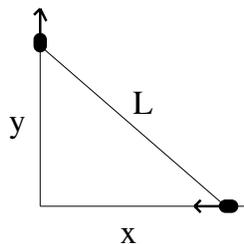
Hint: write the equation in the form

$$x^2 + [y(x)]^2 = 1$$

(where we have indicated that  $y$  depends on  $x$ ). Now differentiate each term with respect to  $x$ .

3. Consider the function  $y = g(x) = x^{1/n}$  where  $n > 1$  is an integer. [Note: this is the inverse function for  $f(x) = x^n$ .] Rewrite this in the form  $y^n = x$  and use implicit differentiation to find the derivative of  $f(x)$ .

4. In the figure shown here, one person walks towards the corner at the rate 1m/s and the other walks away at rate 2 m/s. The distances of the individuals from the corner at time  $t$  are  $x(t)$  and  $y(t)$ . At what rate is  $L$  changing at the instant when  $x = y = 10\text{m}$ ?



5. Find the slope of the tangent line at the point  $(1,1)$  on the curve

$$x^2 + xy + y^2 = 3$$

Hints: it may help you to first indicate that  $y = y(x)$  depends on  $x$ , and then to differentiate each term in the equation. you will need to use the product rule on this problem.

6. Consider the circle given by  $x^2 + y^2 = 1$ , and the parabola

$$y = ax^2 - b.$$

For what values of the constants  $a, b$  does the parabola touch the circle at two points?