(1) The sum of two positive number is 20. Find the numbers if the product of the square of one and the cube of the other is a maximum.

(2) Find the cylinder of maximal volume that would fit inside a sphere of radius $R$. 
(3) Flying birds experience two types of drag forces as they fly. Each type of force depends differently on the speed, $v$ of the bird. The total drag force is

$$F(v) = Av^2 + \frac{B}{v^2}, \quad A, B > 0.$$ 

The constants $A, B$ depend on the size, shape, and other properties of the bird. At what speed $v$ is the drag minimized? What is the minimal drag?

What is the absolute maximum drag that the bird experiences for $-2 \leq v \leq 2$?

(4) A certain population of rodents is exposed to predation by a cat. Suppose that the density of rodents is $x$. The net growth rate of the rodent population, $F(x)$ is given by (birth rate minus rate of loss to predation):

$$F(x) = rx - K \frac{x}{a+x}, \quad r, a, K > 0.$$ 

For what value of $x$ is the net growth rate smallest? Under what condition(s) is there such a minimal net growth rate? If the number of rodents is known to be in the range $0 \leq x \leq P$, find the absolute maximum net growth rate.

(5) A rectangle of side lengths $L_1$ and $L_2$ is to be made into a box with no top by cutting out the corners and folding up 4 sides. What is the largest volume that this box can hold?