1. Consider the differential equation \( \frac{dy}{dt} = a - by \)

(a) What function \( y(t) \) would satisfy this equation and would correspond to a constant solution?

(b) Draw a slope field for this equation in the case of \( a = 1, b = 1 \). Add some solution curves.

(c) What do you expect happens to \( y(t) \) after a long time?


4. Consider a spherical cell and supposed that the density of the cell is \( \rho = 1 \text{ gm/cm}^3 \) (similar to that of water).

(a) Assume the mass of the cell is \( m = \rho V \). Suppose the mass of the cell changes as the cell grows or shrinks. Use related rates to connect the derivative \( dm/dt \) to the date of change of cell radius \( dr/dt \). (Hint: recall that the volume of a sphere is \( V = \frac{4}{3}\pi r^3 \)).

(b) The mass of the cell can increase as the cell absorbs nutrients. Cell mass can also decrease through metabolism that converts nutrients into metabolic energy. Let us denote the absorption and consumption rates by \( A \) and \( C \) (in units of mass/time). Write down a differential equation relating the rate of change of cell mass to \( A \) and \( C \).

(c) Make appropriate assumptions about how \( A \) and \( C \) depend on the cell size, volume, and/or surface area. (You may want to reread Section 1.2 of the Book.

(d) Using all of the above, what is the differential equation that describes the rate of change of the radius of the cell? (Note: your differential equation should be expressed in terms of the radius and various constants. (Not in terms of \( V \) or \( S \)).

(e) Based on (d), describe the qualitative behaviour of the cell size, that is predict whether cell radius increases, decreases, or stays constant, depending on various initial conditions.


6. (Review) Find the cone of greatest volume that fits inside a sphere of radius \( R \). (Note that the volume of a cone of height \( h \) and base radius \( r \) is \( V = \frac{1}{3}\pi r^2 h \).