Math102 Section 110

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Worksheet for Nov 14, 2017

1. Consider a tub of water initially at 60^oC in a room whose constant ambient temperature is $E = 21^o$C. After 5 min, the water has cooled down to 55^oC. When will the water reach the temperature of 44^oC?

2. (a) Solve the same problem as above, namely Newton's Law of Cooling, by using Euler's Method on the the differential equation and initial condition

$$\frac{dT}{dt} = -k(T - E), \quad T(0) = T_0.$$

Take the values of the constant in the problem to be $E = 21$, $T_0 = 60$ (degrees C), and $k = 0.0274$ per min. Use a spreadsheet to calculate the temperature over 200 time-steps of size $h = \Delta t = 0.1$ min.

(b) On a separate column, add the exact solution at the same time points. (You computed this in Q1). Compute the error by finding $T_{exact} - T_{Euler}$. Plot the graphs of the approximate and the exact solutions, and add the error to the same graph. (In order to see that error, consider plotting 100 $\times$ Error, so that it shows up on the same scale as the other values.) Comment on what happens to the error over time. Does it increase? decrease? Why?

3. **Optimization Review:** Find the area of the largest rectangle that can be inscribed in the semi-circle of radius a. (See `http://wiki.ubc.ca/Course:MATH102/Question_Challenge`) Problem 1998 December Q3

4. **Related Rates Review:** A convex lens has focal length 5 cm. Let p be the distance between an object and the lens, and let q be the distance between the image and the lens. Then

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

where$f$, the focal length, is constant. If the object is moving away from the lens at 2 cm/sec, how fast is the image moving at the instant that the object is 20 cm from the lens? Is the image moving towards the lens or away from it?

(See `http://wiki.ubc.ca/Course:MATH102/Question_Challenge`) Problem 2002 December Q5