Math102 Section 106, Leah Keshet **Quiz 3, Nov 16, 2017: GROUP STAGE -**

time t (sec)	height (cm)h
5	12
25	10
39	8
57	6
79	4
104	2

Table 1: Data for the height of fluid remaining in the cylindrical container at time t

Student Name	**Student ID**	**email address**

Work in a group of 3-4. Please DO USE laptops/tablets/ electronic devices.

TOTAL POINTS VALUE: 5

[Q1-Q5] Answer by clickers

Question [6] The height $h(t)$ of fluid draining in a cylindrical container with a small hole is described by the differential equation $\frac{dh}{dt} = -k\sqrt{h}, \quad h(0) = h_0$. Here we will use the fact that a solution to this differential equation is

$$h(t) = (\sqrt{h_0} - \frac{1}{2}kt)^2. \tag{1}$$

In an experiment at home with a simple pop bottle, I got the (approximate) data shown on the cover page. It is recommended to put this data on a spreadsheet.

(a) Rewrite the relationship in (1) so that it is linear in t ("transform variables"). (Remark: you did something like this in OSH 4 with exponentials and logs, but consider just taking square-roots of both sides of (1)).

My rewritten ("linear") version of Equation (1) is:

_____ = _____

(b) Use your spreadsheet to find a best-fit line $y = ax + b$ to the (transformed) data. Use that line to find the value of the constant k and the initial height of the fluid, $h(0) = h_0$.

Bestfitline: $y =$ _____

$h_0 =$ _____ , $k =$ _____ .

(c) According to the prediction of the model, at what time would the fluid level have dropped to $h = 0$?

At $t =$ _____

Solution to Q5

(b) The function is

$$h(t) = (\sqrt{h_0} - \frac{1}{2}kt)^2 \quad \Rightarrow \quad \sqrt{h} = \sqrt{h_0} - \frac{1}{2}kt$$

Hence, plotting $\sqrt{h}$ against t should give a line of slope $-k/2$ and intercept $\sqrt{h_0}$.

The answers to the next questions depend on whether you used the data given on the cover page, or the data collected in class

For the data on the cover sheet:

time t (sec)	height (cm)h
5	12
25	10
39	8
57	6
79	4
104	2

Table 2: Data I got earlier for the height of fluid remaining in the cylindrical container at time t

For the data in Table 2 (c) The best fit line to the transformed data (using excel linear fit) is

$$y = 3.6321 - 0.021x$$

Hence $\sqrt{h_0} = 3.6321$ so $h_0 = 3.6321^2 = 13.19215$ and the value of the constant $-k$ is the slope so $k = 2 \times 0.021 = 0.042$

(d) The height of the water at time $t = 0$ was $h_0 = 3.19215^2 = 13.19215$? The container is empty when

$$h(t) = 0, \quad \Rightarrow \quad \sqrt{h_0} - \frac{1}{2}kt = 0 \quad \Rightarrow \quad t = 2\sqrt{h_0}/k = \frac{2 \cdot 3.19215}{0.042} = 173 \text{ min.}$$

For the data collected in class:

time t (sec)	height (cm)h
0	14
13.46	12
26.33	10
40.8	8
58.35	6
79	4
106	2

Table 3: Data collected in class for the height of fluid remaining at time t

If you used the data in Table 3 (c) The best fit line to the transformed data (using excel linear fit) is

$$y = 3.7436 - 0.0221x$$

Hence $\sqrt{h_0} = 3.7436$ so $h_0 = 3.7436^2 = 14.014541$ and the value of the constant $-k$ is the slope so $k = 2 \times 0.0221 = 0.0442$

(d) The height of the water at time $t = 0$ was $h_0 = 3.7436^2 = 14.01454096$? The container is empty when

$$h(t) = 0, \quad \Rightarrow \quad \sqrt{h_0} - \frac{1}{2}kt = 0 \quad \Rightarrow \quad t = 2\sqrt{h_0}/k = \frac{2 \cdot 3.7436}{0.0442} = 169.4 \text{ min.}$$