Math102 Section 106

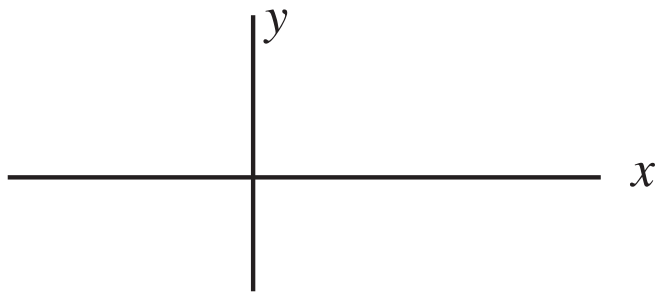
Instructor: L. Keshet Your email address: _____

Quiz 2, Oct 5, 2017: INDIVIDUAL STAGE

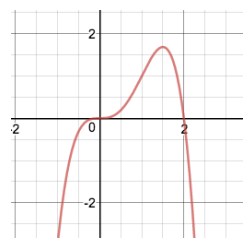
Work on your own. No notes, books, calculators, or electronic devices.

TOTAL POSSIBLE POINTS: 10 Pts

(Q1) Draw a rough sketch of the function $y = f_0(x) = 2x^3 - x^4$. (No need to use calculus.)



Solution to Q1 We can construct the sketch by considering which power function dominates near the origin ($y \approx 2x^3$) and which dominates for large x ($y \approx -x^4$). Then we smoothly connect the curve to get a rough sketch. We also note that $f(x) = 0$ at $x = 0$ and at $x = 2$.



POINTS: 1 FOR OVERALL SHAPE, 1 FOR GOING THROUGH (0,0)- TOTAL 2 PTS

(Q2) Find the equation of the tangent line to the graph of the function $y = f(x) = 2x^3 - x^4 + 4$ at the point $x = 1$. (NOTE: *the function is similar but not quite the same as in* Q1.)

$y =$_____

Solution to Q2 The function is $y = f(x) = 2x^3 - x^4 + 4$ and its derivative is $f'(x) = 6x^2 - 4x^3$. At the point $x = 1$, we have $f(1) = 2 - 1 + 4 = 5$, so the point of tangency is $(1, 5)$. The slope of the tangent line is $f'(1) = 6 - 4 = 2$. So the equation of the tangent line is obtained from

$$\frac{y-5}{x-1} = 2 \quad \Rightarrow \quad y = 2(x-1) + 5 = 3 + 2x.$$

So the equation of the tangent line is $y = 3 + 2x$.

POINTS: point of tangency - 1pt, slope of TL - 1pt, Eqn. of TL 1 pt- TOTAL 3 PTS

(Q3) Find the point at which the tangent line you just found intersects the x axis. (Leave answer as a fraction.)

x_{inters} =_____

Solution to Q3 That point is obtained from setting $y = 0$ on the tangent line equation, so $y = 0 = 3 + 2x$. $x = -3/2 = -1.5$ (decimal value not required).

POINTS: setting $y = 0$ - 1 PT; solving and correct answ - 2 pts; TOTAL 3 Pts

(Q4) You want to find a zero of the function $y = f(x) = 2x^3 - x^4 + 4$. Suppose you start with the initial guess $x_0 = 1$. Compute the improved value x_1 obtained using Newton's Method. (Leave your answer as a simple fraction of the form a/b).

x_1 =_____

Solution to Q4 It should be fairly obvious that the answer is the same as the answer to Q2, since Newton's Method simply produces the value x_1 as the point at which the tangent line at x_0 crosses the x axis. However, for those who did not notice this, and used the formula, it would be

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{5}{2} = -3/2.$$

POINTS: TOTAL 2 Pts (part marks for right Newton's formula but wrong calculation)