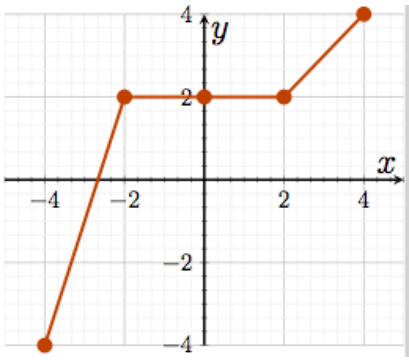
10 points total Math 102 Section 110 Solutions to Quiz # 1 September 21, 2017

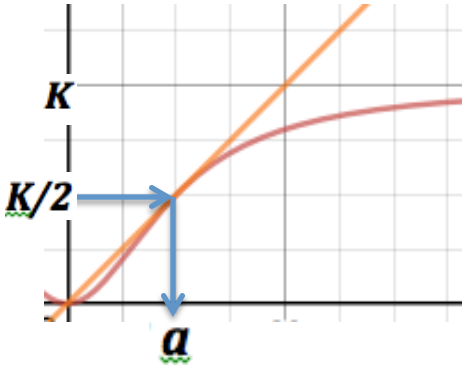


(1) Over what interval does the function depicted in the graph have the greatest average rate of change?

We look for the interval such that a line between its two endpoints on the graph has largest slope. We find that this is the interval **[-4,-2]** (where the slope is 6/2=3). It can be seen that the slope of lines connecting any other pairs of points is shallower.

Answer: √ $-4 \le x \le -2$ √

The average rate of change = 6/2=3 √



(2) The sketch is shown at right. Important aspects: *P(x)* is a Hill function and has a "parabolic shape" close to $x = 0$. √

It has horizontal asymptote K √ and half-max value a. The function *G(x)* is a straight line of slope r. √

For the case $\boldsymbol{K = 2\,a\,r}$ the graphs intersect at only 1 point. (at $x=a$) √

(See Lecture 2.2, and aphid-ladybug problem in OpenBook).

(3) To make the function continuous, we have to match its values from the left and the right at each of the endpoints. At x=2 we plug 2 into x=a+bx to get 2=a+2b.
At x=4 we plug 4 into a+bx=6 to get a+4b=6.
We now have two equations in two unknowns:

$$f = \begin{cases} x & x \le 2 \\ a + bx & 2 < x < 4 \\ 6 & 4 \le x \end{cases}$$

$a+2b=2$

$a+4b=6$

Solving these we get

2b=4 → b = 2 and then $a=2-4 = -2$.

The solution is thus √ $a = -2$, $b = 2$. √ √

The function is shown on the right.