(1) Over what interval does the function depicted in the graph have the greatest average rate of change?

We look for the interval such that a line between its two endpoints on the graph has largest slope. We find that this is the interval \([-4,-2]\) (where the slope is \(6/2=3\)). It can be seen that the slope of lines connecting any other pairs of points is shallower.

Answer: \(\sqrt[\text{\(\checkmark\)}} -4 \leq x \leq -2 \) \(\sqrt[\text{\(\checkmark\)}}

The average rate of change = \(6/2=3\) \(\sqrt[\text{\(\checkmark\)}}

(2) The sketch is shown at right. Important aspects: \(P(x)\) is a Hill function and has a “parabolic shape” close to \(x = 0\). \(\sqrt[\text{\(\checkmark\)}}

It has horizontal asymptote \(K\) \(\sqrt[\text{\(\checkmark\)}} \) and half-max value \(a\). The function \(G(x)\) is a straight line of slope \(r\). \(\sqrt[\text{\(\checkmark\)}}

For the case \(K = 2 \ a \ r\) the graphs intersect at only 1 point.

(at \(x=a\) \(\sqrt[\text{\(\checkmark\)}}\)

(See Lecture 2.2, and aphid-ladybug problem in OpenBook).

(3) To make the function continuous, we have to match its values from the left and the right at each of the endpoints. At \(x=2\) we plug 2 into \(x=a+bx\) to get \(2=a+2b\).

At \(x=4\) we plug 4 into \(a+bx=6\) to get \(a+4b=6\).

We now have two equations in two unknowns:

\[
\begin{align*}
a+2b &= 2 \\
a+4b &= 6
\end{align*}
\]

Solving these we get

\(2b=4 \rightarrow b = 2\) and then \(a=2-4 = -2\).

The solution is thus \(\sqrt[\text{\(\checkmark\)}} a = -2, \ b = 2. \sqrt[\text{\(\checkmark\)}}

The function is shown on the right.