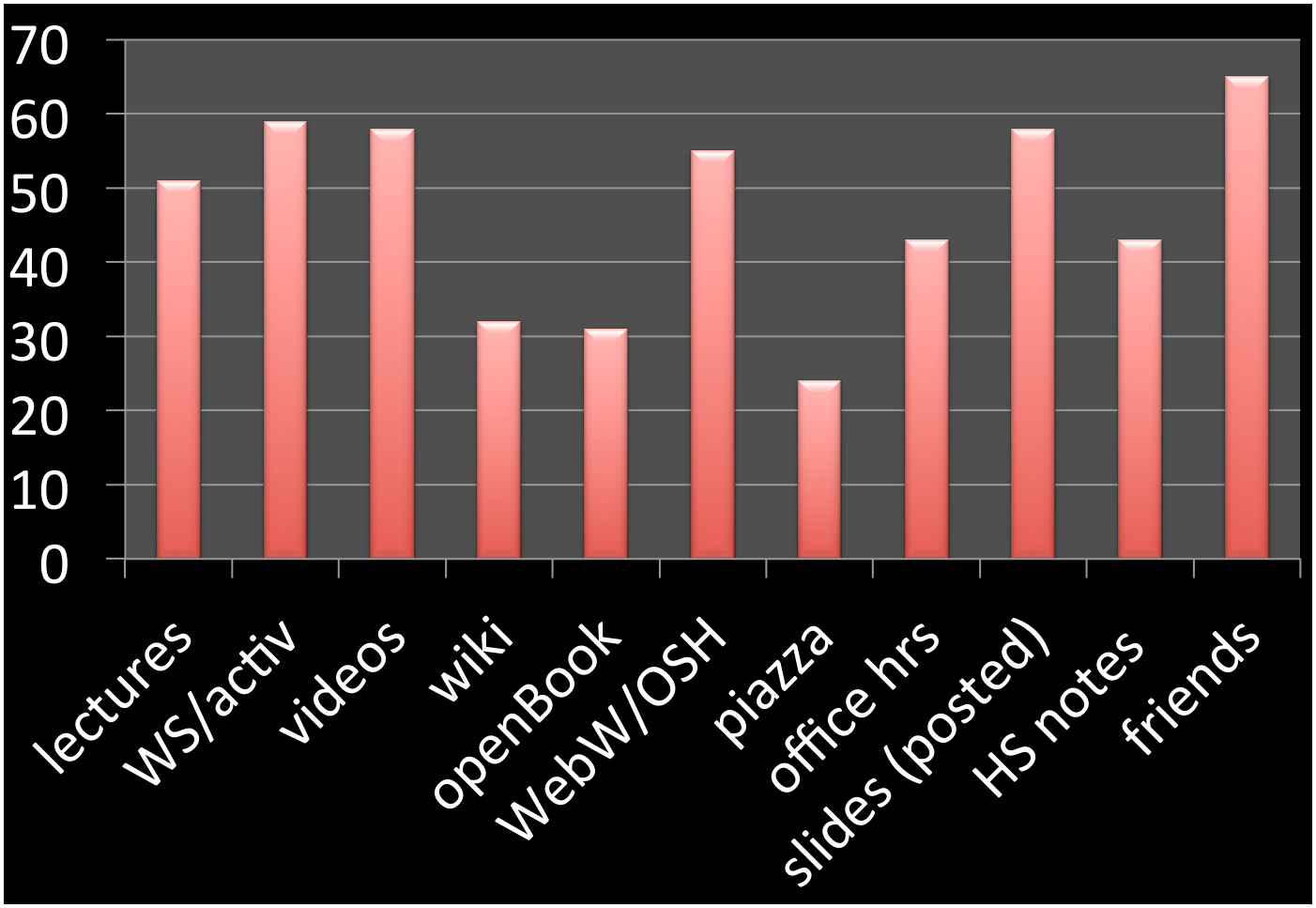
Exponential growth and differential equations

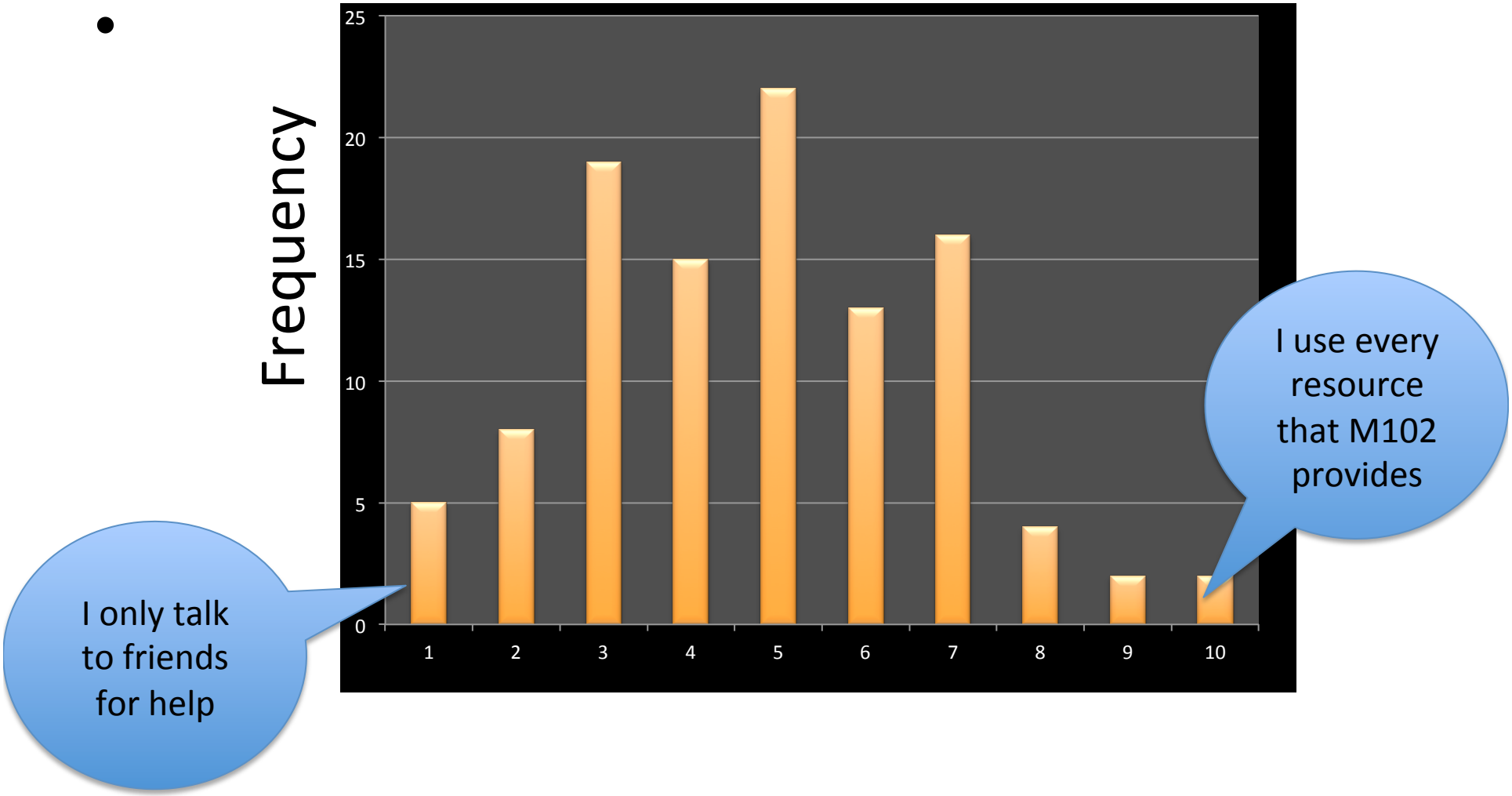
But first.. Thanks for the feedback!

Feedback about M102

- Which of the following do you find useful?



How many resources students typically use

- 

Common suggestions

- Do more harder problems just like the ones on the midterms and the exam (40-50 responses).

Really?

- Let us clear up a few misconceptions first..

Comparison of some problems we did

IN CLASS

How to outrun a cheeta

- Gazelle has constant velocity: find its position
- Determine when they meet by equating positions

Exam-type question

ON MIDTERM

Hare and Fox problem

- Fox has constant velocity; find its position
- Determine when they meet by equating positions

Comparison of some problems we did

IN CLASS

Aphid laydybug problem

- Solve an equation of the form

$$K\frac{x^n}{a^n+x^n} = rx$$

ON MIDTERM

Hare and Fox problem

- Solve an equation of the form

$$\frac{12t}{3+t} = vt$$

Exam-type question

Comparison of some problems we did

IN CLASS

Rodent population problem (Worksheet Oct 10)

- Find minimum of the function

$$F(x) = rx - K\frac{x}{a+x},$$

ON MIDTERM

Hare and Fox problem

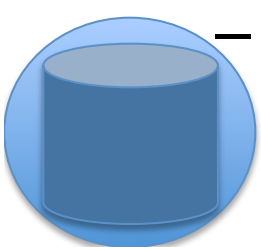
- Find where the following function is increasing/decreasing

$$\frac{12t}{3+t} = vt$$

Comparison of some problems

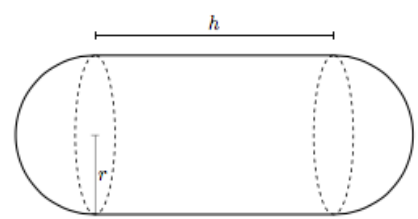
IN CLASS

- Find cylinder of max volume that fits inside sphere (WS Oct 10):
  - Write down Volume V
  - Formulate constraint (constant sphere R)
  - Eliminate on variable
  - Optimize V(r)
  - Check the type of CP



ON MIDTERM

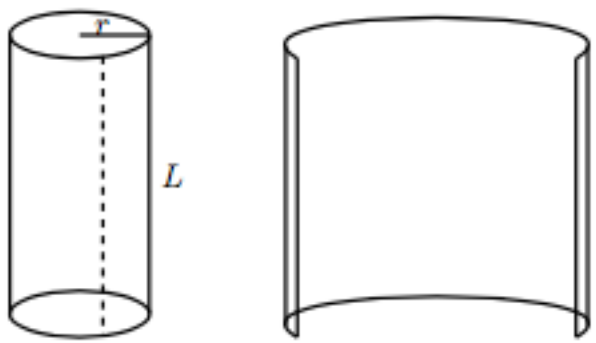
- E coli volume optimization problem
  - Write down Volume V
  - Formulate constraint (constant surface S)
  - Eliminate on variable
  - Optimize V(r)
  - Check the type of CP



Comparison of some problems

IN BOOK

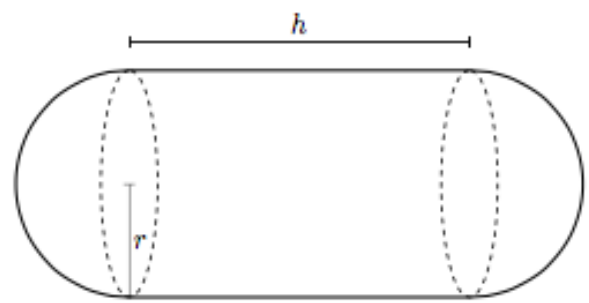
A cylindrical cell with minimal surface area



See P 148 Sec 7.2

ON MIDTERM

- E coli volume optimization problem



Comparison of some problems

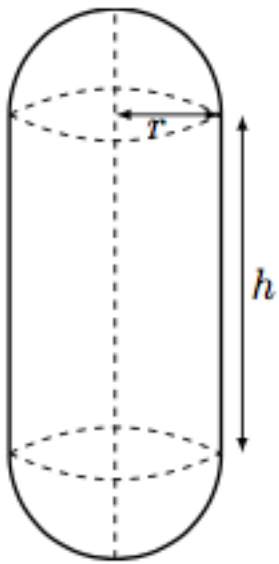
IN BOOK

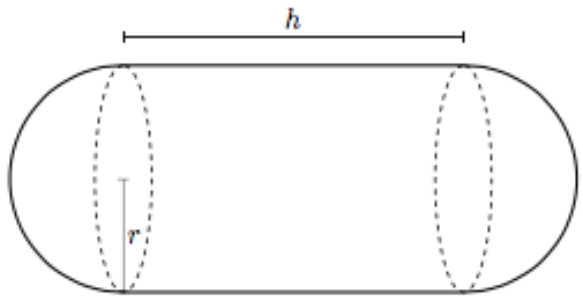
- E coli volume optimization problem

ON MIDTERM

- E coli volume optimization problem

See Problem 16.15





More comparison

IN CLASS

- Clicker problems on critical points, Inflection points, derivatives, etc:
  - MathGame Q # 25
  - Lect 5.1

ON MIDTERM

- MC # 6 “the hard one”

6. Which of the following statements is true for **any** differentiable function f? If more than one statement is true, choose (E).

   A. If the graph of $f'(x)$ looks like the line with equation $y = 3x$ when zooming in at $x = 0$, then $f(x)$ has a local minimum at 0.

   B. If $f''(\pi) = 0$, then the concavity of $f(x)$ changes at π.

   C. If $f'(x)$ is continuous, then $f''(x)$ exists.

   D. If the function $f(x)$ considered on the interval $2 \leq x \leq 3$ has an absolute minimum over this interval, then $f'(x)$ has a zero.

   E. More than one of the above statements is true.

“Really hard questions”

- .. Yes there were a few “tricky bits”.. OK..

Feedback, cont'd

- Do more step-by-step solutions
- More detail on problems/solutions
- More guidance through worksheets
- More time on each question

(ABOUT 13 such responses)

good ideas, and easy to fix!

Feedback

- Display clicker answers immediately after a clicker question

good idea, and easy to fix!

- Solving problems on the board or document camera, rather than just showing solutions on the slides

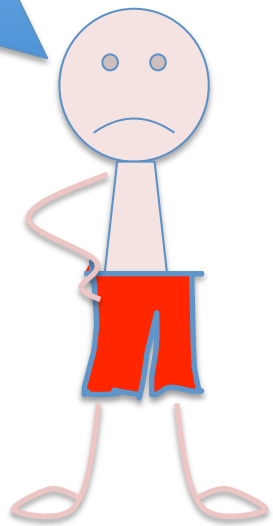
good idea, and easy to fix!

Miscellanious Feedback

- “I want [the instructor] to show me how to solve problems.. I don’t want to try to figure it out on my own”

That's a bit like this:

- "Coach, if you don't mind, I'd rather just watch you teach me how to swim, instead of getting into the water myself"



Some expectations..

- Mastery of a subject requires thinking, analysing, practicing, questioning, and persistence (not just rote, not just WebWork)
- You must learn how to learn..
- In class we can demo tools and ideas, but you have to take responsibility for owning them, using them creatively in new ways.. We can not “teach you how to solve every problem”..

Feedback

- Talk louder
- It is hard to focus in this class

Feedback

- Talk louder
- please be quieter and more respectful
- It is hard to focus in this class

My idea for a fix

- Time to discuss

- Time to focus and be quiet

Observation about exponential functions

Exponential function

- Last time:
- The derivative of e^x is e^x

- (by the chain rule) The derivative of e^{kx} is $k\ e^{kx}$

<u>Let us see what this implies</u>

A new kind of equation

The function $y = f(x) = e^x$ has the same function as its derivative. Hence the function f(x) satisfies the equation

$$dy/dx = y$$

The derivative is the same as the original function

What if it depends on time?

The function $y = f(t) = e^t$ has the same function as its derivative. Hence the function $f(t)$ satisfies the equation

$$dy/dt = y$$

The derivative is the same as the original function

One more slight change

The function $y = f(t) = e^{kt}$ has the derivative ke^{kt}. Hence the function $f(t)$ satisfies the equation

$$dy/dt = k\,y$$

The derivative is some constant times the original function

“Differential equation”

- An equation that involves (one or more) derivative of a function (and possibly the function itself) is called a differential equation.

- For example: $dy/dt = k\,y$ is a differential equation (DE).
- The function $y = f(t)$ is a solution if it satisfies this equation.
- We have just seen that $y = e^{kt}$ satisfies this DE.

Solution to a differential equation

- $y=e^{kt}$ is a solution to the differential equation:

$$dy/dt = ky$$

Is that the only solution that works?

Solution to a differential equation

Which of the following functions satisfy the differential equation

$$\frac{dy}{dt} = -ky$$

(A) $y = e^{-kt}$, (B) $y = 10e^{-kt}$, (C) $y = 2e^{-kt}$,

(D) $y = Ce^{-kt}$, (E) All of these,

Solution to a differential equation

Which of the following functions satisfy the differential equation

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Solutions to a differential equation

The function $y=Ce^{kt}$ is a solution to the differential equation $dy/dt = ky$ for any constant C!

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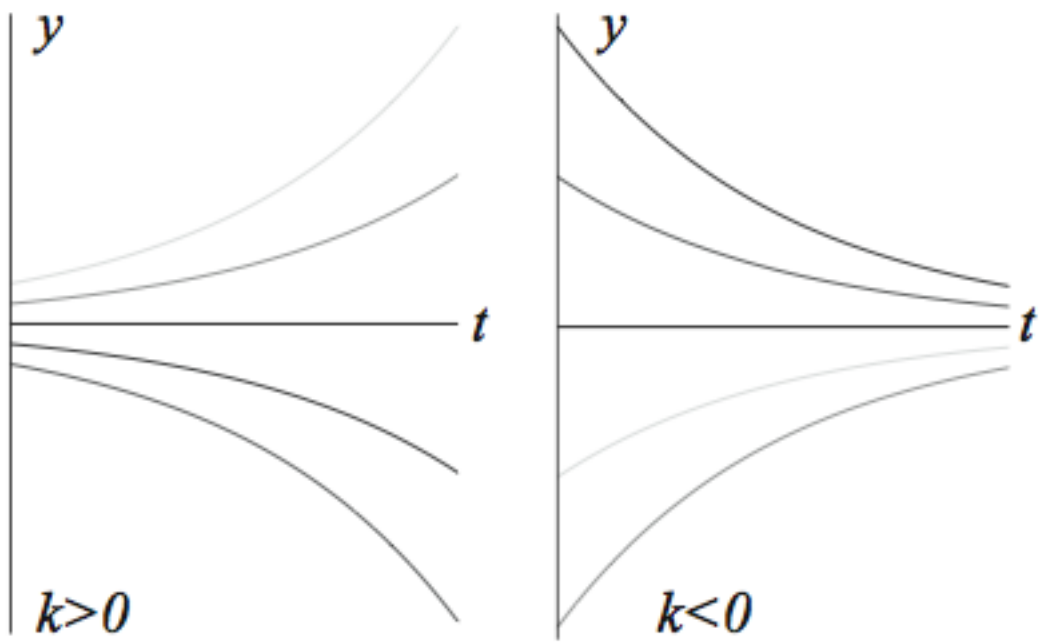

Initial condition

A function satisfies the the differential equation

$$\frac{dy}{dt} = -5y$$

We are told that at time t = 0 the value of y is y(0) = 3. Then the function is

$$\text{(A) } y = 3e^{-5t}, \quad \text{(B) } y = 5e^{-3t}, \quad \text{(C) } y = 15e^{-5t},$$

$$\text{(D) } y = 3^{-5t}, \quad \text{(E) } y = 5^{-3t},$$

Initial condition

A function satisfies the the differential equation

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We are told that at time t = 0 the value of y is y(0) = 3. Then the function is

$\boxed{\text{(A) } y = 3e^{-5t}}$, (B) $y = 5e^{-3t}$, (C) $y = 15e^{-5t}$,

(D) $y = 3^{-5t}$, (E) $y = 5^{-3t}$,

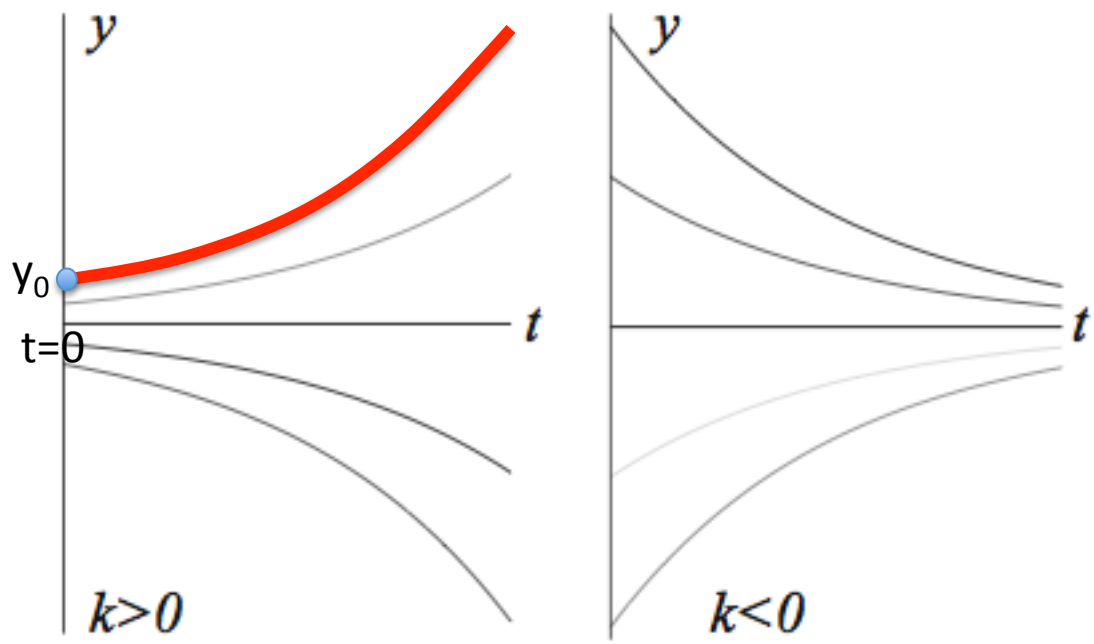
Solutions to a differential equation

The function $y=Ce^{kt}$ is a solution to the differential equation $dy/dt = ky$ for any constant C!

We need more information (such as the state at time *t=0*) to specify the value of the constant C.

Solution to "initial value problem" (diff'l eqn + IC)

The function $y=y_0e^{kt}$ is the solution to the differential equation $dy/dt = ky$ and IC $y(0)=y_0$



Differential equations in Exponential Population growth

UBC Math 102

Balance equations

Previously, we asked when two processes exactly balance

- E.g.:

nutrient absorption rate = nutrient consumption rate

Aphid birth rate = rate of aphid mortality due to predation

(0) But.. What if..

What is the two things DON'T BALANCE??

In that case we get **change**

e.g. if aphid birth rate > mortality rate then

(A) The population increases

(B) The population decreases

(C) The problem is not defined

(D) Not sure what happens

(0) But.. What if..

What is the two things DON'T BALANCE??

In that case we get **change**

e.g. if aphid birth rate > mortality rate then

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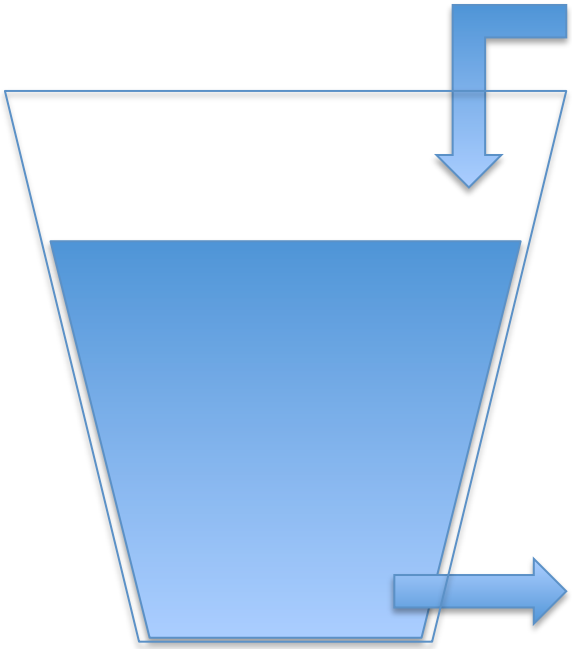
(C) The problem is not defined

(D) Not sure what happens

A differential equation is

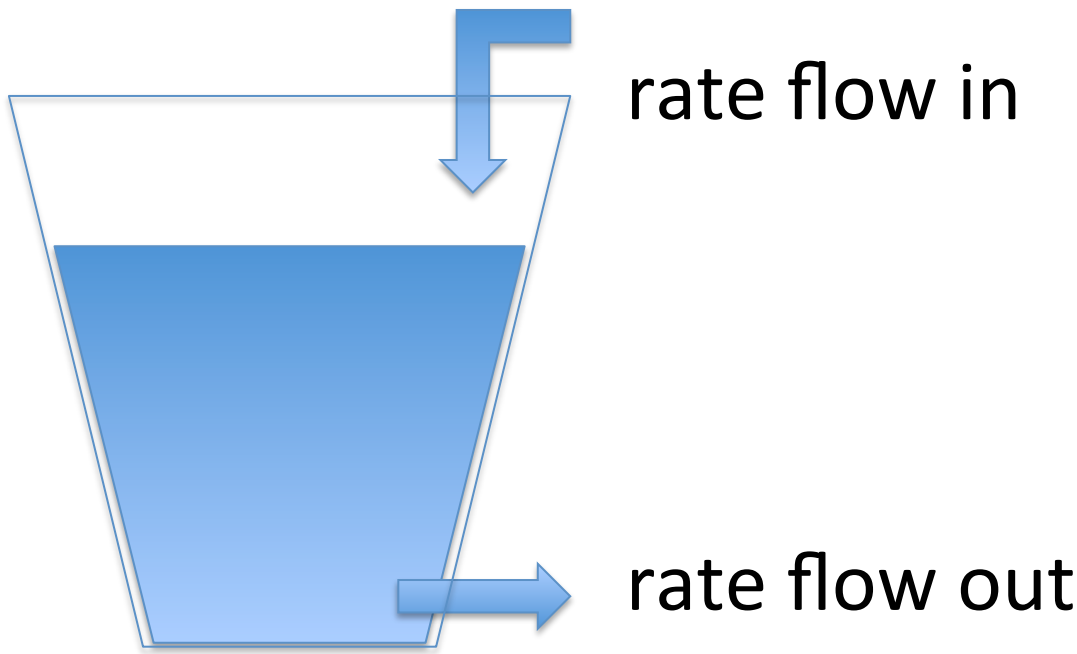
- .. A statement that allows us to track those changes



A differential equation:

"Rate of change of amount

= rate flow in – rate flow out"



A differential equation

- Describes the rate of change of some "state variable"

Example: t = time

N(t) = population at time t

Rate of change of N(t) ($\Delta N/\Delta$time)	=	Rate of births (num/time)	-	Rate of deaths (num/time)	+	Rate of immigration (num/time)

Units

- Notice: units of each term are the same

Rate of change of N(t) **(number/time)**	=	Rate of births **(num/time)**	-	Rate of deaths **(num/time)**	+	Rate of immigration **(num/time)**

(1) What is this term?

Rate of change of N(t) **(number/time)**

(A) N(t)

(B) N/t

(C) dN/dt

(D) None of the above

(E) Not sure

(1) What is this term?

Rate of change of N(t) **(number/time)**

(A) N(t)

(B) N/t

(C) dN/dt

(D) None of the above

(E) Not sure

Simple Example

- Per capita birth rate = r (per unit time)
- Per capita mortality = m (per unit time)
- Immigration rate = I (number per unit time)

Assume r, m, I constants > 0

(2) We obtain the differential equation for population growth:

(A) $dN/dt = r - m + I$

(B) $dN/dt = r + m + I$

(C) $dN/dt = (r-m) N + I$

(D) $dN/dt = (m-r) N$

(E) $dN/dt = (r-m) I$

(2) We obtain the differential equation for population growth:

(A) $dN/dt = r - m + I$

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(C) $dN/dt = (r-m)\,N + I$

(D) $dN/dt = (m-r)\,N$

(E) $dN/dt = (r-m)\,I$

The differential equation:

- We obtain

Rate of change of N(t) **(number/time)**	=	Rate of births **(num/time)**	-	Rate of deaths **(num/time)**	+	Rate of immigration **(num/time)**

$$dN/dt = r N - m N + I$$

$$\boxed{dN/dt = (r-m) N + I}$$

What does this equation predict?

Case 1: r>0, m>0, I = 0 (No immigration)

$$\boxed{dN/dt = \underbrace{(r-m)}_{k} N}$$

Let k = (r-m) ← "net growth rate"

Note: k could be 0, positive or negative.

(3) What does this equation predict?

Case 1: $r>0$, $m>0$, $I = 0$ (No immigration)

$$dN/dt = k\,N \qquad \text{where } k = (r-m)$$

Then if $k>0$ we expect the population would:

(A) Increase (B) decrease (C) no change

(D) Not sure

(3) What does this equation predict?

Case 1: $r>0$, $m>0$, $I = 0$ (No immigration)

$$dN/dt = k\,N \qquad \text{where } k = (r-m)$$

Then if $k>0$ we expect the population would:

(A) Increase (B) decrease (C) no change

(D) Not sure

Intuitively: $k>0$

- If $r>m$ → birth rate > mortality rate N ↑
- Also $k=0$ $(r = m)$ → birth = mortality
- $k<0$ $(r < m)$ → birth < mortality N↓

But we can do even better! We can say exactly what the population level will be at any later time, if we know its size at time $t=0$

What function satisfies

The Diffl Eqn: $dN/dt = kN$

And Initial value: $N(0)=N_0$

Looking for a function of time N(t) whose derivative is proportional to itself!

(4) What function satisfies

The Diffl Eqn: $dN/dt = kN$

And Initial value: $N(0)=N_0$

(A) $N(t)=kt$

(B) $N(t)= (1/2)kt^2$

(C) $(1/2)\, k\, N^2$

(D) $N(t)= e^{kt}$

(E) $N(t)= N_0\, e^{kt}$

(4) What function satisfies

The Diffl Eqn: $dN/dt = kN$

And Initial value: $N(0)=N_0$

(A) $N(t)=kt$

(B) $N(t)= (1/2)kt^2$

(C) $(1/2)\, k\, N^2$

(D) $N(t)= e^{kt}$

$\boxed{\text{(E) } N(t)= N_0\, e^{kt}}$

We know how to solve this initial value problem!

Differential equation: $dN/dt = k N$ $k = (r-m)$

Some initial condition: $N(0)=N_0$

The above pair of equations (a differential equation together with an initial condition) is called an Initial Value Problem (IVP)

→ Solution: $N(t) = N_0 e^{kt}$

Check:

Note: given a function, we can always test whether it satisfies a differential equation!

Check that $N(t)= N_0 e^{kt}$ is a solution to the diff'l eqn (DE) and initial condition(IC)

$dN/dt = k N$ $\qquad$ $N(0)=N_0$

Check:

Differentiate the function $N(t) = N_0 e^{kt}$

We get:

$$\frac{dN}{dt} = N_0 \frac{d}{dt} e^{kt} = N_0 \cdot k e^{kt}$$

$$= k\left(N_0 e^{kt}\right) = k \cdot N$$

also $N(0) = N_0$: $N(0) = N_0 e^0 = N_0$

So indeed, this function satisfies the DE and IC

Sketch our result

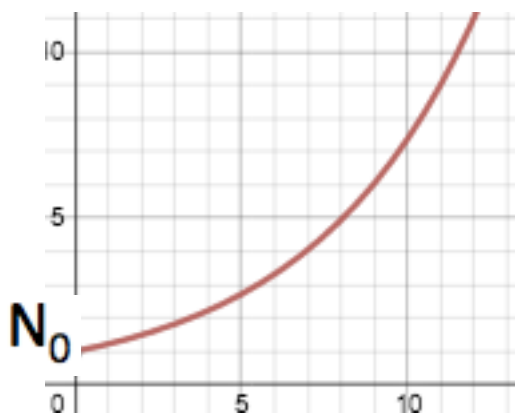
Draw a rough sketch of the function $N(t)= N_0 e^{kt}$ (where $k=r-m$) for

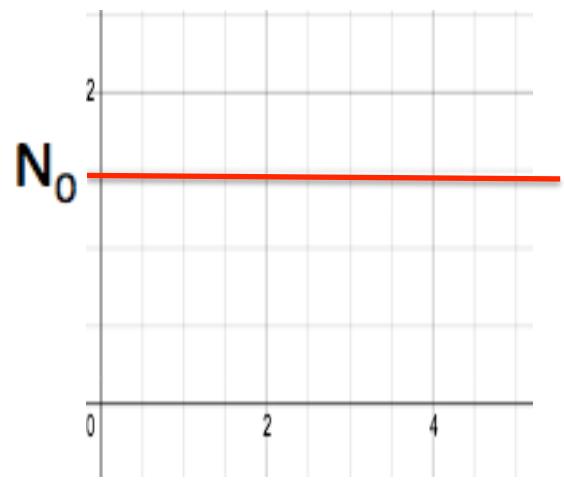
(1) $r>m$, (2) $r<m$, (3) $r=m$.

Sketch our result

Draw a rough sketch of the function $N(t)= N_0 e^{kt}$ (where k=r-m) for

(1) k>0, (2)k<0, (3) k=0.



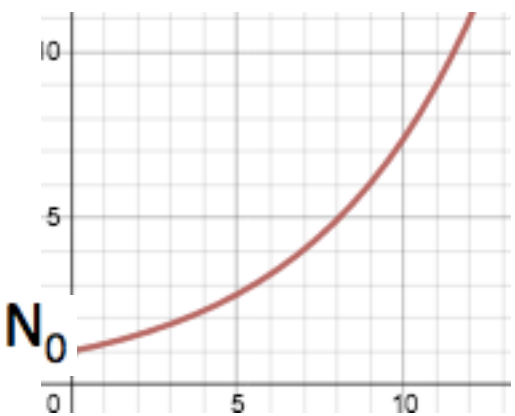


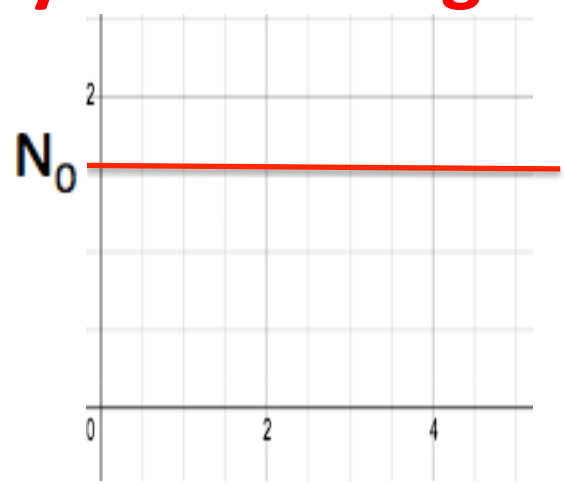
Sketch our result

Draw a rough sketch of the function $N(t)= N_0 e^{kt}$ (where k=r-m) for

(1) k>0, (2)k<0, (3) k=0.

expon'tial growth **expon' decay** **no change**





What's this all about???

- Early part of term: given a function, we learned how to find its derivative (and used that to help graph, find max, mins, etc)

- Now: Given some information about the derivative, we are trying to find the function!
- (Not as simple as just finding an antiderivative because the function and its derivative are all mixed up together.)

Case 2: Immigration in Europe

In Europe, birth rates are lower than mortality, but there is a constant rate of immigration, I

$$dN/dt = I - \mu N \quad \text{where } \mu = m - r > 0$$

Case 2: Immigration (I)

$$dN/dt = I - \mu N \qquad I, \quad \mu = m - r > 0$$

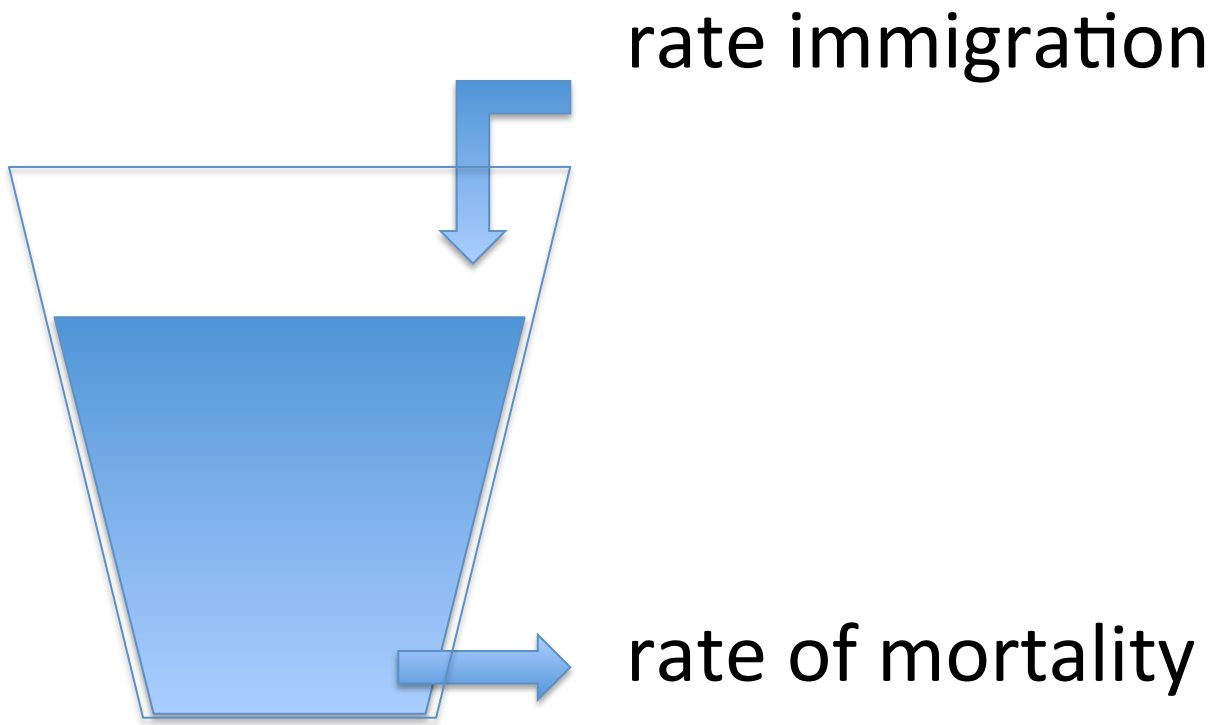
Belongs to a group of diff'l eqns such as

$$dy/dt = a - by \qquad a, b > 0$$

What it says:

dN/dt = rate immigr – rate mortality”

$$dN/dt = I - \mu N$$



When is there a balance between immigration and mortality?

$$dN/dt = I - \mu N$$

Is there a constant population level N that satisfies this equation?

When is there a balance between immigration and mortality?

$$dN/dt = I - \mu N$$

When $I = \mu N$ then the two balance

and then $$N = I/\mu$$

(and also $dN/dt = 0$)

When is there a balance between immigration and mortality?

$dN/dt = I - \mu N$

When $I = \mu N$ then the two balance

and then $N = I/\mu$

(and also $dN/dt = 0$)

We refer to that situation as a **Steady State**

(there is no change overall, even though both processes continue)

What if immigration and mortality don't balance?

Then N(t) will change!

$$dN/dt = I - \mu N$$

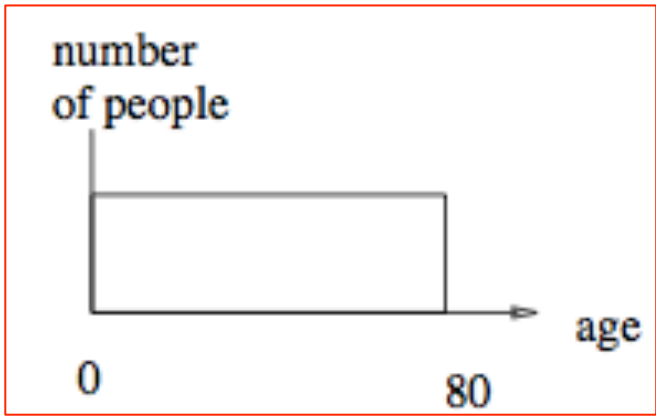
Population will either increase or decrease. It turns out that it will move towards the steady state

$$N = I/\mu$$

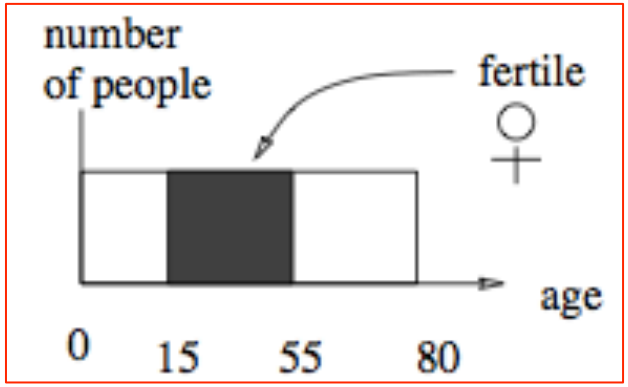
Now back to human population explosion:

Overall human population on Planet Earth:

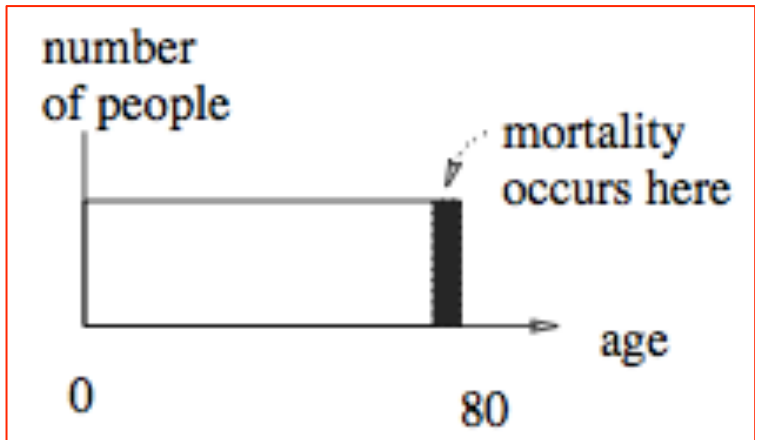
- Simple assumption about fertility



- Simple assumption about mortality rate



See P233
Section 11.2



Rough estimates for r and m:

- Per capita birth rate $r \approx 0.025$ per year

- Per capita mortality rate $m \approx 0.0125$ per year

$\rightarrow \quad k=(r-m) = 0.0125$ /year

Doubling time:

$dN/dt = k\,N$, with

$k = (r-m) = 0.0125\ /yr$

$\rightarrow \quad N(t) = N_0\, e^{\,0.0125\, t}$

How long till the population DOUBLES?

(5) Doubling time:

$dN/dt = k\,N$, with

$k = (r-m) = 0.0125$ /yr

$\rightarrow \quad N(t) = N_0\, e^{0.0125\, t}$

How long till the population DOUBLES?

(A) 0.0125 years (B) $e^{0.0125}$ years

(C) 10 years (D) ln(2)/0.0125 years

(E) ln(0.0125)/2

(5) Doubling time:

$dN/dt = k\,N$, with

$k = (r-m) = 0.0125$ /yr

$\rightarrow \quad N(t) = N_0\, e^{0.0125\, t}$

How long till the population DOUBLES?

(A) 0.0125 years (B) $e^{0.0125}$ years

(C) 10 years $\boxed{\text{(D) } \ln(2)/0.0125 \text{ years}}$

(E) $\ln(0.0125)/2$

(5) Doubling time:

$$N(t) = N_0 \, e^{0.0125\, t}$$

Doubling time = $\ln(2)/0.0125 = 55.5$ years

Predicted human population growth:

$dN/dt = k\,N$, with

$k = (r-m) = 0.0125\ /yr$

$N(t) = N_0\, e^{\,0.0125\,t}$

If we start with 6 billion now, how many in 100 ys?

(6) Rough estimate

According to our model ("Malthusian growth) in 100 years there will be how many humans?

(A) 7 billion

(B) 10 billion

(C) 15 billion

(D) 20 billion

(E) 40 billion

(6) Rough estimate

According to our model ("Malthusian growth) in 100 years there will be how many humans?

(A) 7 billion

(B) 10 billion

(C) 15 billion

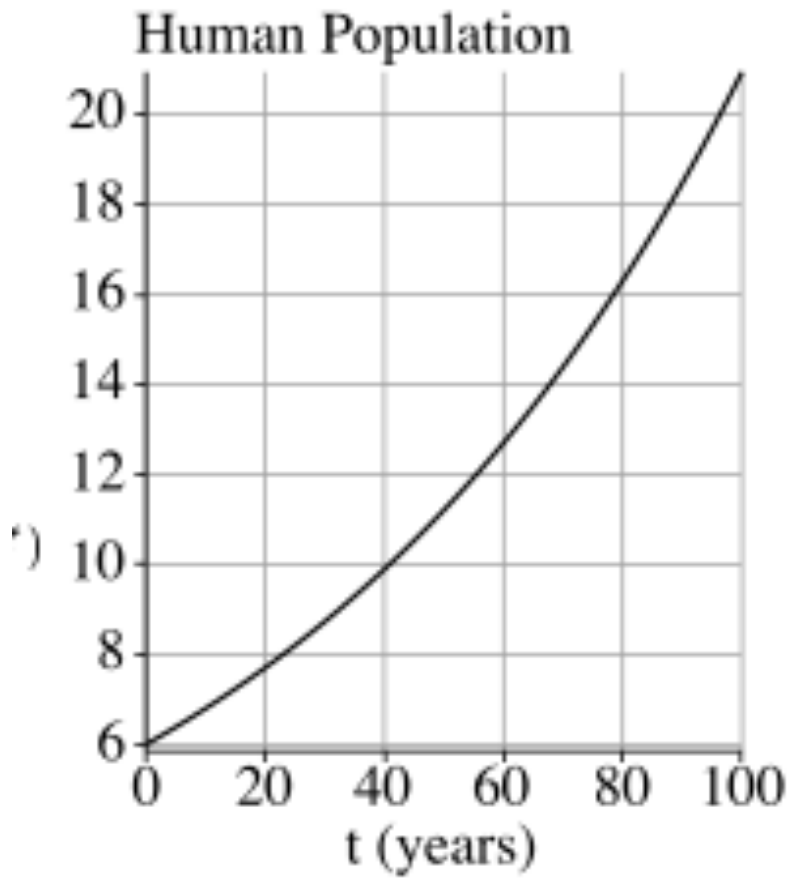
(D) 20 billion

(E) 40 billion

Predicted human population growth:

$N(t) = 6\, e^{0.0125\, t}$ billion

If we keep growing, at t= 100 ys we will be around 20 billion strong!



Problems to test your skills

Final exam Q

Question 3: Blood Alcohol Level (BAL), the amount of alcohol in your blood stream (here represented by $B(t)$), is measured in milligrams of alcohol per 100 milliliters of blood. At the end of a party (time $t = 0$), a drinker is found to have $B(0) = 0.08$ (the legal level for driving impairment), and after that time $B(t)$ satisfies the differential equation

$$\frac{dB}{dt} = -kB$$

where k is a positive constant that represents the rate of removal of alcohol from the blood-steam by the liver.

(a) If the drinker had waited for three hours before driving, (until $t = 3$) his BAL would have dropped to 0.04. Determine the value of the rate constant k (specifying appropriate units) for this drinker.

(b) According to this model, how much longer would it take for the BAL to drop to 0.01?

Final Exam Q:

4. The populations of two species of rodents are found to satisfy

$$y_1(t) = 1000\,e^{-0.2t}$$
$$y_2(t) = 50\,e^{0.5t}$$

(a) What differential equations and initial conditions govern the population of each species?

(b) At what time is the population of species 1 twice as large as that of species 2?

Solutions from last time

Exam question:

iii. (3 points) Let f^{-1} be the inverse function of $f(x)$. Assume $f(0) = 1$ and $f'(0) = 2$. Find the tangent line $y = mx + b$ to $f^{-1}(x)$ at 1.

Solution:

The point (0,1) is on the graph of f so (1,0) is the corresponding point on the graph of f^{-1}.

also $f'(0) = 2 \Rightarrow (f^{-1})'(1) = \frac{1}{2}$ (recipr.)

T.L. through $(1,0)$, slope $\frac{1}{2}$

$$\frac{y-0}{x-1} = \frac{1}{2} \qquad y = \frac{1}{2}(x-1).$$

A midterm problem "puzzler"

Compute the derivative of $y=f(x) = x^x$

Hint: take **ln** of both sides

The derivative of y=f(x) = x^x

$$y = x^x$$

$$\ln y = \ln(x^x) = x \ln x$$

$$\frac{d}{dx} \ln y = \frac{d}{dx}(x \ln x)$$

$$\frac{d \ln y}{dy} \cdot \frac{dy}{dx} = 1 \ln x + \frac{1}{x} \cdot x$$

$$\frac{1}{y} \frac{dy}{dx} = \ln x + 1$$

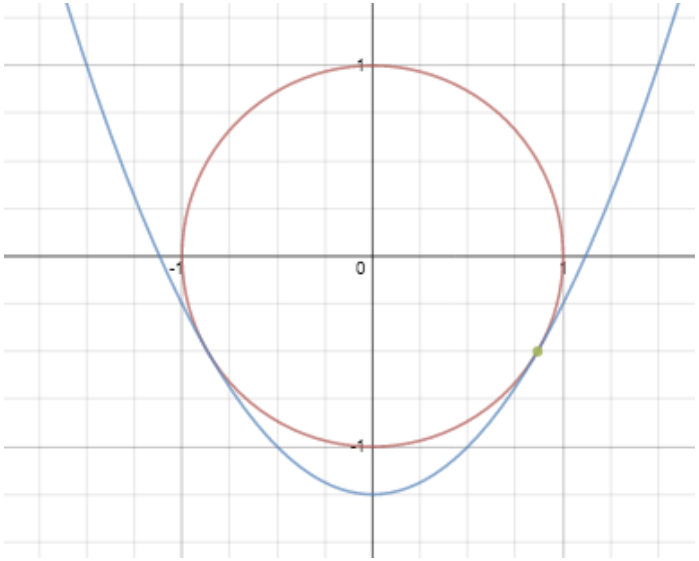
$$\frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$$

From last worksheet

Consider the circle given by $x^2 + y^2 = 1$, and the parabola

$$y = ax^2 - b.$$

For what values of the constants a, b does the parabola touch the circle at two points?



The plan

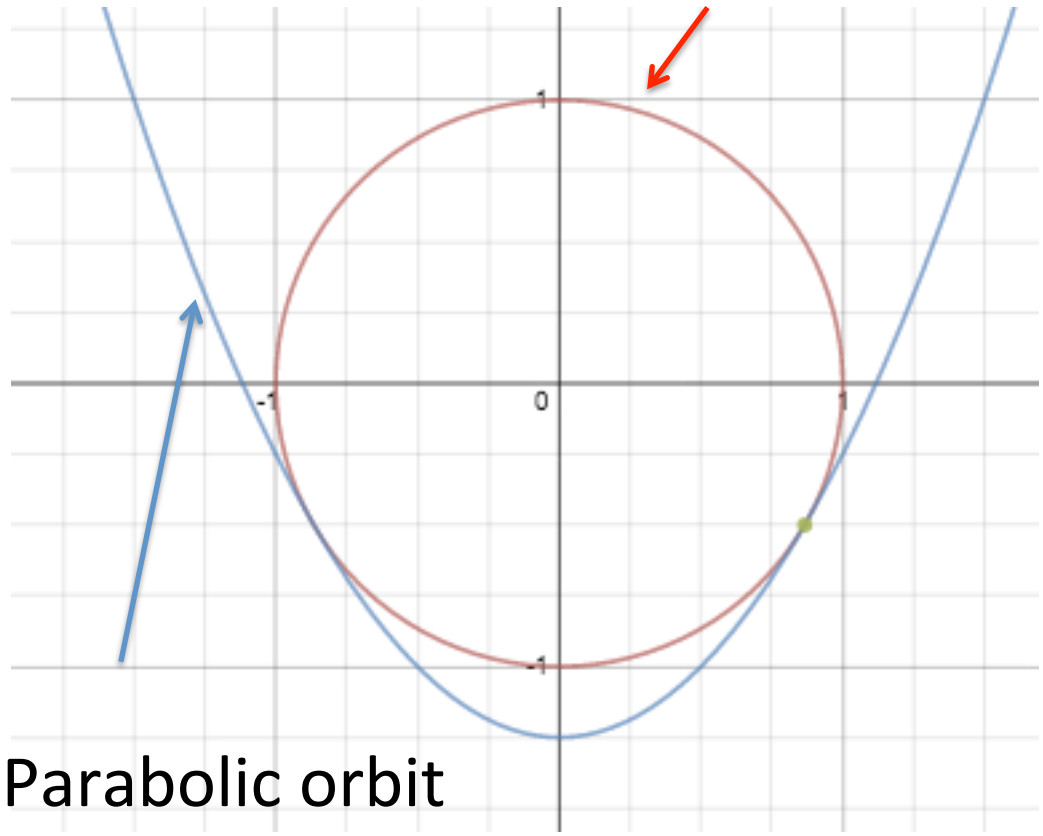
Circle

$$x^2 + y^2 = 1$$

Parabola

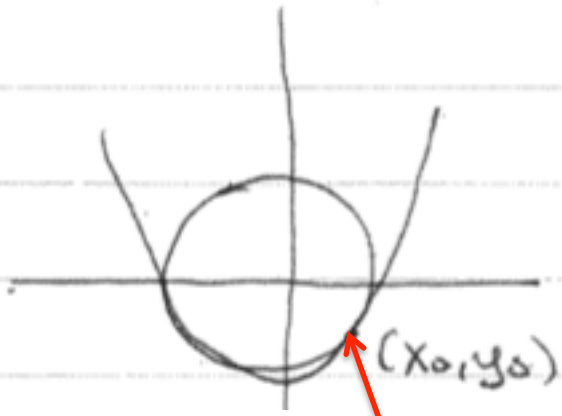
$$y = ax^2 - b$$

circular orbit



Parabolic orbit

Solution: Set up the problem



$y = ax^2 - b$ parabola

$x^2 + y^2 = 1$ circle of radius 1

Label the point(s) at which the orbits intersect

Equate slopes of tangent lines

$$x^2 + y^2 = 1 \qquad\qquad y = ax^2 - b$$

Find dy/dx for both the curves and get the equation that results by setting them equal at the points (x_0, y_0).

What are possible solutions?

Find points of intersection

- point of intersection (x_0, y_0) satisfies both eqns

parabola $\quad y_0 = a x_0^2 - b \Rightarrow x_0^2 = \dfrac{y_0 + b}{a}$

Sub in

circle $\quad x_0^2 + y_0^2 = 1 \Rightarrow$

$$\frac{y_0 + b}{a} + y_0^2 = 1 \Rightarrow a y_0^2 + y_0 + b = a$$

Get quadratic eqn in y_0 → $\quad a y_0^2 + y_0 + (b - a) = 0$

solve this quadratic eqn for y_0:

$$y_0 = \frac{-1 \pm \sqrt{1 - 4a(b-a)}}{2a}$$

This is the y coordinate of that point. We can get the x coordinate from the above

Get slopes of tangents to each curve

Implicit differentiation used for slope of circle

slope of parab: $\frac{dy}{dx} = 2ax$

$\parallel$

slope of circle $2 \cdot x + 2y \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

Equate slopes at meeting point

at pt of tangency $\quad 2a x_0 = -\frac{x_0}{y_0}$

There are two possible cases:

$\Rightarrow$ either $x_0 = 0$

case ①:

on parab: $y_0 = -b$

on circle $y_0 = \pm 1$

$\Rightarrow b = \pm 1$

or

case ②

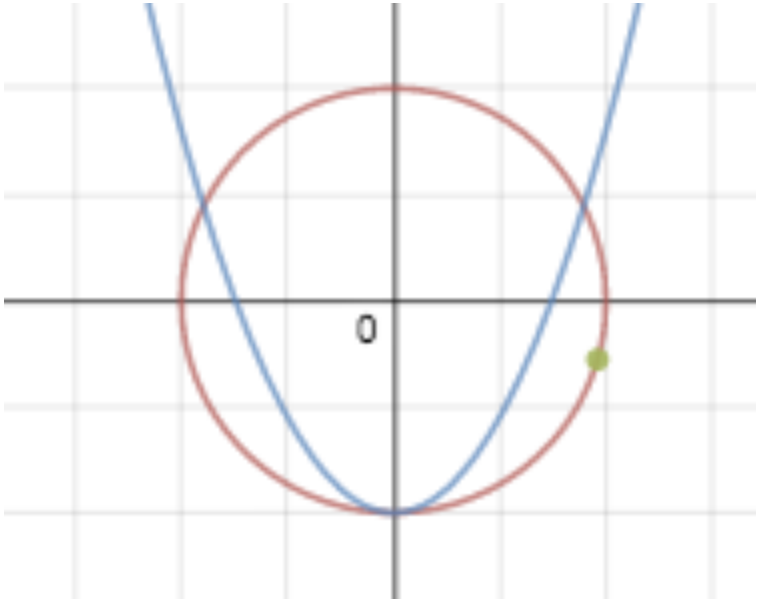
$2a = -\frac{1}{y_0}$

$y_0 = -\frac{1}{2a}$

Case 1

This case is less interesting – it turns out to have one single solution at the tip or the base of the circle:

Only one tangent point, at the bottom of the parabola



Case 2

We have two simultaneous equations for y_0, which both have to be satisfied. Using both equations we get

$$y_0 = \frac{-1}{2a} \overset{\downarrow}{=} \frac{-1 \pm \sqrt{1-4a(b-a)}}{2a}$$

$$\Downarrow$$

$$\cancel{-1} = \cancel{-1} \pm \sqrt{1-4a(b-a)}$$

$$0 = \sqrt{1-4a(b-a)}$$

$$0 = 1-4a(b-a)$$

Requirement for such point to exist

The above gives us a relation between a and b (parabola parameters) that must be satisfied, that we can solve for b in terms of a:

$$0 = 1 - 4a(b-a)$$

$$1 = 4a(b-a)$$

$$\frac{1}{4a} = b-a$$

$$b = \frac{1}{4a} + a$$

Designing a parabolic orbit

$$y = ax^2 - b$$

We should set b= $\left(a + \frac{1}{(4a)}\right)$

This is a requirement for the parabola to intersect the circle at two tangent points.

We can test this in Desmos.

Try this yourself!

1 $x^2 + y^2 = 1$

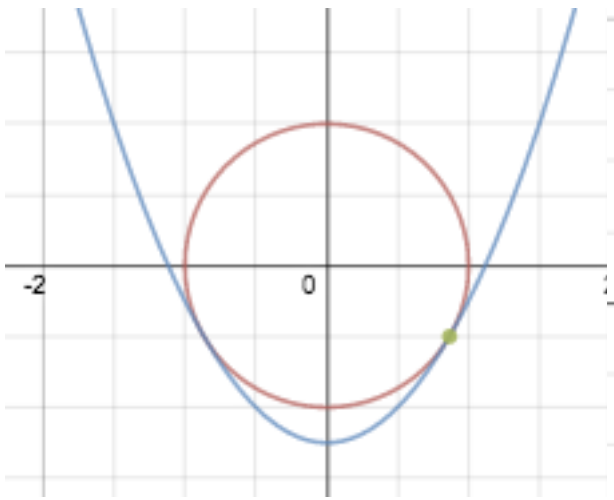
2 $y = ax^2 - \left(a + \frac{1}{(4a)}\right)$

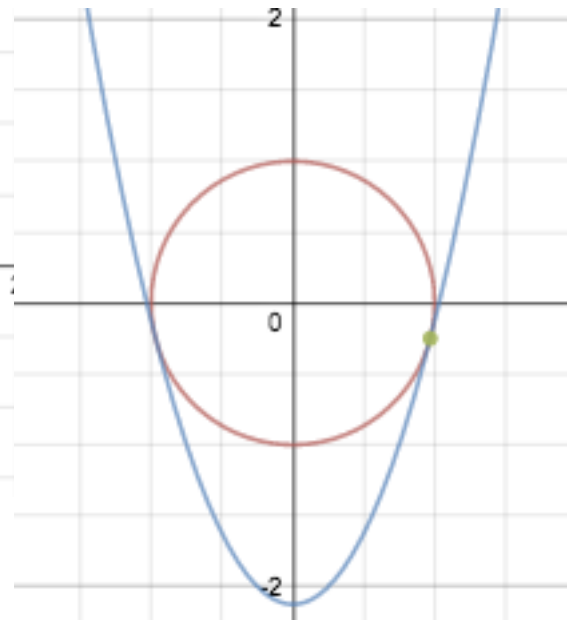
4 $a = 1$

0

$a = 2$

0





We use the value of b that we just computed and a slider for a

Are there any special constraints on *a*?

- Exploration on desmos suggests that we have to be a little careful! If *a* is too small, the two curves fail to meet at all!

- Check it out. So what else did we have to notice?

Conclusion

For the given circular orbit of the supply craft, we need to select a parabolic orbit of the space station of the form:

$$y = ax^2 - \left(a + \frac{1}{(4)}\right)$$

For some sufficiently large constant *a*.

(There are other considerations, such as speeds and time of contact that we will leave for NASA rocket scientists).

Differential equations for exponential growth and decay

(preview of next time)

Differential equation

- The function e^x satisfies the equation:

$$\frac{dy}{dx} = e^x = y$$

A differential equation is an equation linking a function and its derivatives.

Solution to a differential equation

- We say that $y=e^x$ is a **solution** to the differential equation:

$$dy/dx = y$$

Solution to a differential equation

- We want to use these facts in time-dependent systems. Hence the independent variable will usually be t for time rather than x.

Solution to a differential equation

- We say that $y=e^{kt}$ is a solution to the differential equation:

$$dy/dt = ky$$

Notice: independent variable t (time)

Solution from last time:

Question 3: Blood Alcohol Level (BAL), the amount of alcohol in your blood stream (here represented by $B(t)$), is measured in milligrams of alcohol per 100 milliliters of blood. At the end of a party (time $t = 0$), a drinker is found to have $B(0) = 0.08$ (the legal level for driving impairment), and after that time $B(t)$ satisfies the differential equation

$$\frac{dB}{dt} = -kB$$

where k is a positive constant that represents the rate of removal of alcohol from the blood-steam by the liver.

$B(t) = B_0 e^{-kt}$ ✓ $\quad B(0) = B_0 = 0.08$ } 1 pts

(a) If the drinker had waited for three hours before driving, (until $t = 3$) his BAL would have dropped to 0.04. Determine the value of the rate constant k (specifying appropriate units) for this drinker.

BAL decreases by 50% after 3 hrs

$\Rightarrow$ half life is 3 hrs $= \tau_{1/2}$ ✓

$k = \frac{\ln 2}{\tau_{1/2}} = \frac{\ln 2}{3} = 0.23$ ✓ per hr ✓ } 4 pts

✓

Solution, cont'd

(b) According to this model, how much longer would it take for the BAL to drop to 0.01?

$$B(t) = 0.08\, e^{-0.23t}$$

$$0.01 = 0.08\, e^{-0.23t}$$

$$\frac{1}{8} = e^{-0.23t}$$

$$\ln 8 = 0.23t$$

$$t = \frac{\ln 8}{0.23} = 9.0$$

Solution from last time:

4. The populations of two species of rodents are found to satisfy

$$y_1(t) = 1000\,e^{-0.2t}$$
$$y_2(t) = 50\,e^{0.5t}$$

(a) What differential equations and initial conditions govern the population of each species?

$$\frac{dy_1}{dt} = -0.2\,y_1 \qquad y_1(0) = 1000$$

$$\frac{dy_2}{dt} = 0.5\,y_2 \qquad y_2(0) = 50$$

Solution cont'd

(b) At what time is the population of species 1 twice as large as that of species 2?

$$y_1(t) = 2y_2(t)$$

$$1000\, e^{-0.2t} = 100\, e^{0.5t}$$

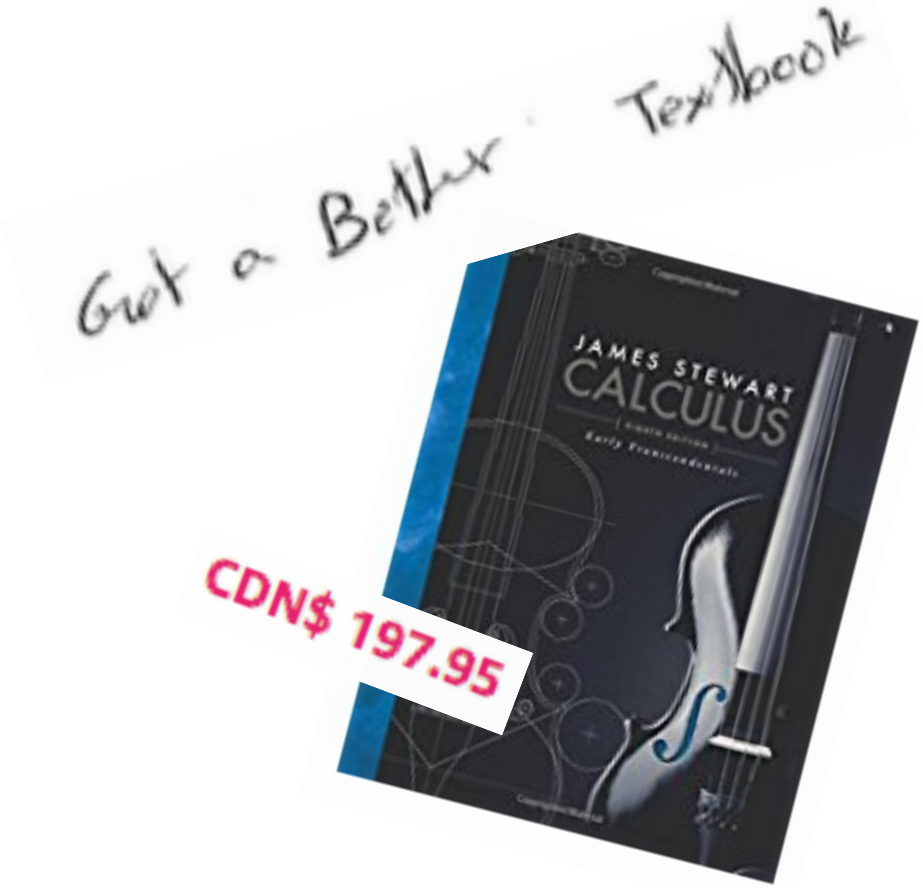
$$\frac{1000}{100} = e^{(0.5+0.2)t}$$

$$10 = e^{0.7t}$$

$$\ln 10 = 0.7t$$

$$t = \frac{\ln 10}{0.7}$$

Miscellanious feedback



Hmm… OK, which one wold you prefer?