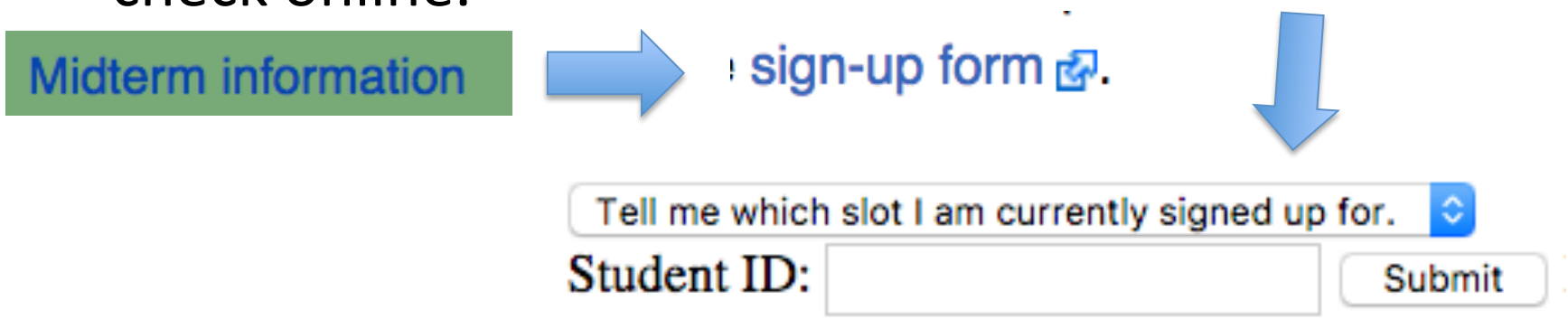
Midterm Info

- Please show up to the place and time you're registered for
  - The signup form will tell you where to go!!
  - check online.



- If you have a conflict, you should have gotten an email from Dr. Yeager.

Chain Rule

Continued

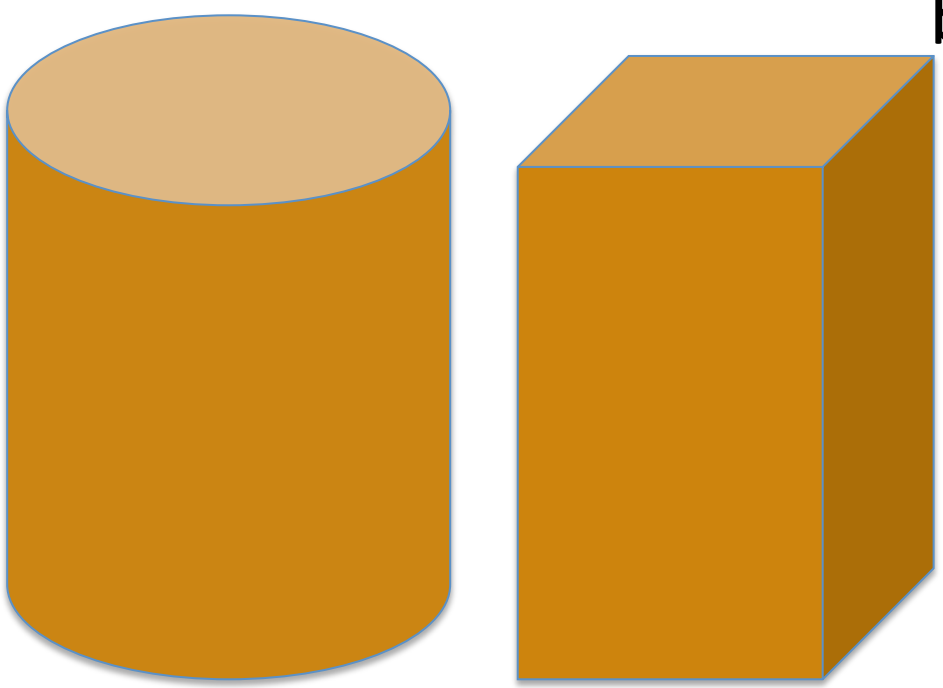
From last week's worksheet

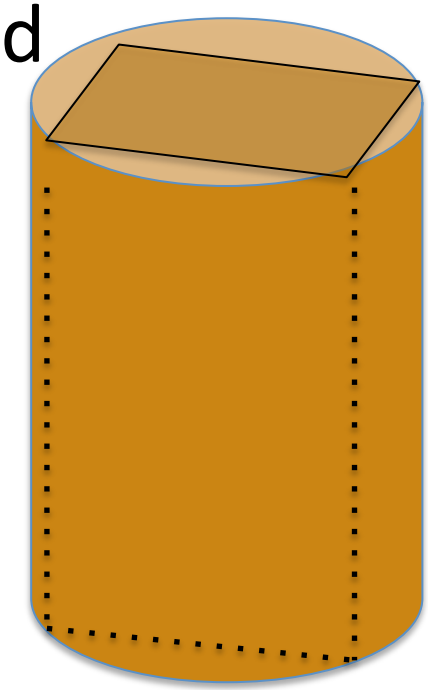
Most economical wooden beam

Exam-type question

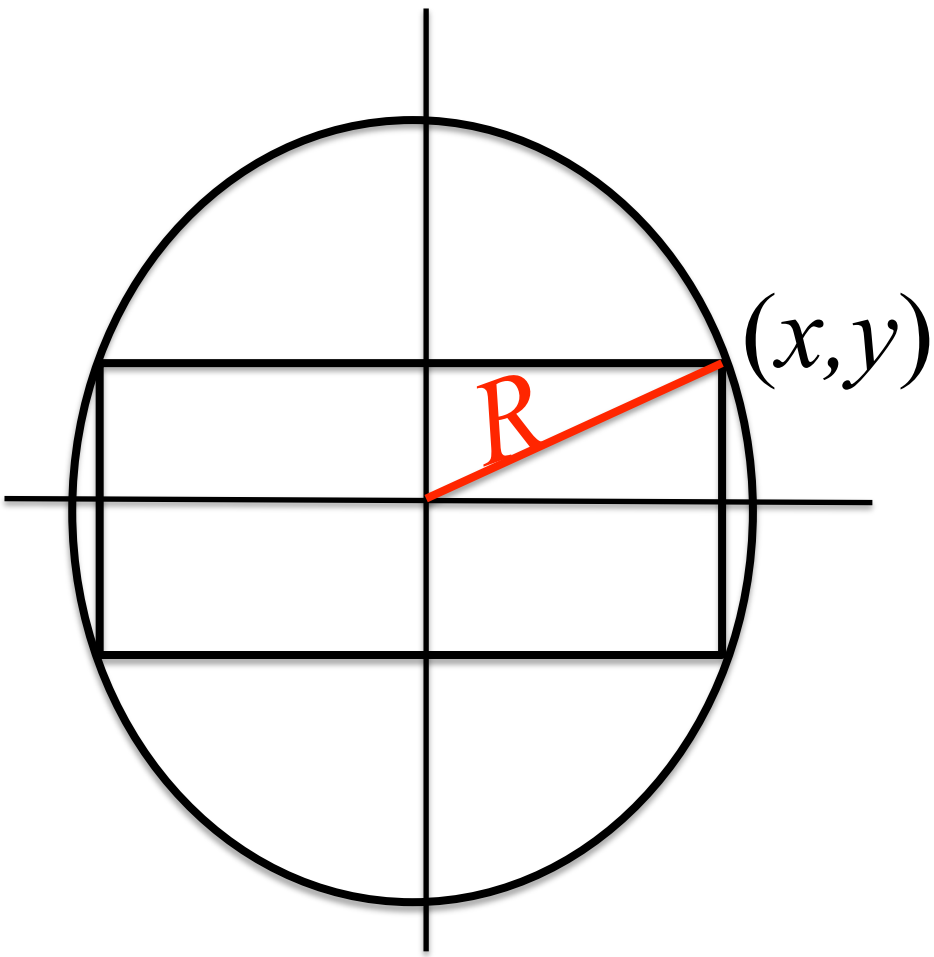
Tree trunk → beam

The height of the beam is fixed





The geometry

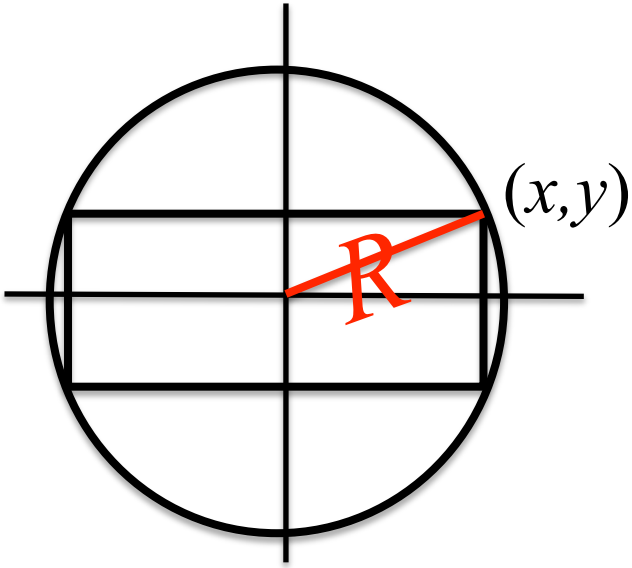


Draw and label a diagram! Since height fixed, largest beam volume determined by largest cross-sectional area.

Restated problem:

Find the largest rectangle that fits inside a circle of radius R

Constraint: $x^2 + y^2 = R^2$



Maximize: $A = 4xy$

Steps in the solution

Constraint: $$x^2 + y^2 = R^2 \quad \Rightarrow \quad y = \sqrt{R^2 - x^2}.$$

Eliminate y:

R is constant

$$A = 4xy = 4x\sqrt{R^2 - x^2} = 4(R^2x^2 - x^4)^{1/2}.$$

Find critical points of A(x): CHAIN RULE

$$\frac{dA}{dx} = 4 \cdot \frac{1}{2}(R^2x^2 - x^4)^{-1/2}\left(2R^2x - 4x^3\right)$$

$$= 2\frac{2R^2x - 4x^3}{(R^2x^2 - x^4)^{1/2}} = 0.$$

Solution, cont'd

- Solve for CP: $$2\frac{2R^2x - 4x^3}{(R^2x^2 - x^4)^{1/2}} = 0.$$
- Get: $x = 0$ or $x = R/\sqrt{2}.$
- Type of CP? Easiest to use either 1st derivative test or sketch the function A(x) = $4x\sqrt{R^2 - x^2}$
- (A(x)=0 at x=0,R and A(x)>0 between those points, which is good enough to indicate a local max)

Problem 4

An animal looks for two food types (1 and 2). The probabilities P_1, P_2 of finding food type 1 and 2 depend on the amounts of attention, x, y that are devoted to searching for each food type. Suppose that

$$P_1(x) = x^2, \quad P_2(y) = y^3.$$

The animal has a fixed total attention span, $x + y = 1$. How should it split attention between looking for food type 1 and 2 so as to maximize the total nutritional value V, where

$$V(x) = P_1(x) + P_2(y).$$

See Book for full solution.

Exam-type question

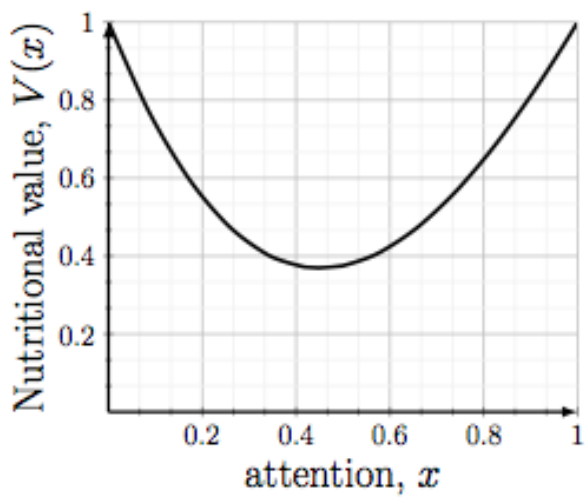
See P 175 "Food choice.."

Note common trap !!

- The nutritional value: $V(x) = x^2 + (1-x)^3.$

This turns out to have a local minimum, not a local maxmum. (It's best NOT to split attention).

Always check the CP!!



Caution !

Chain Rule

Continued: Related Rates

UBC Math 102

Related rates:

- There is an independent variable (time) that all quantities depend on.

$\longrightarrow$ t

“Chain”

Time t → $F(t)$ → $G(F(t))$ → y

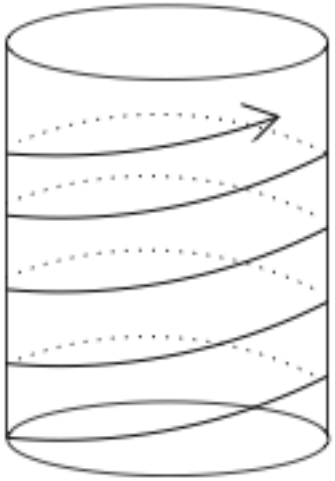
Chain Rule

Time t → $F(t) = u$ → $G(u)$ → y

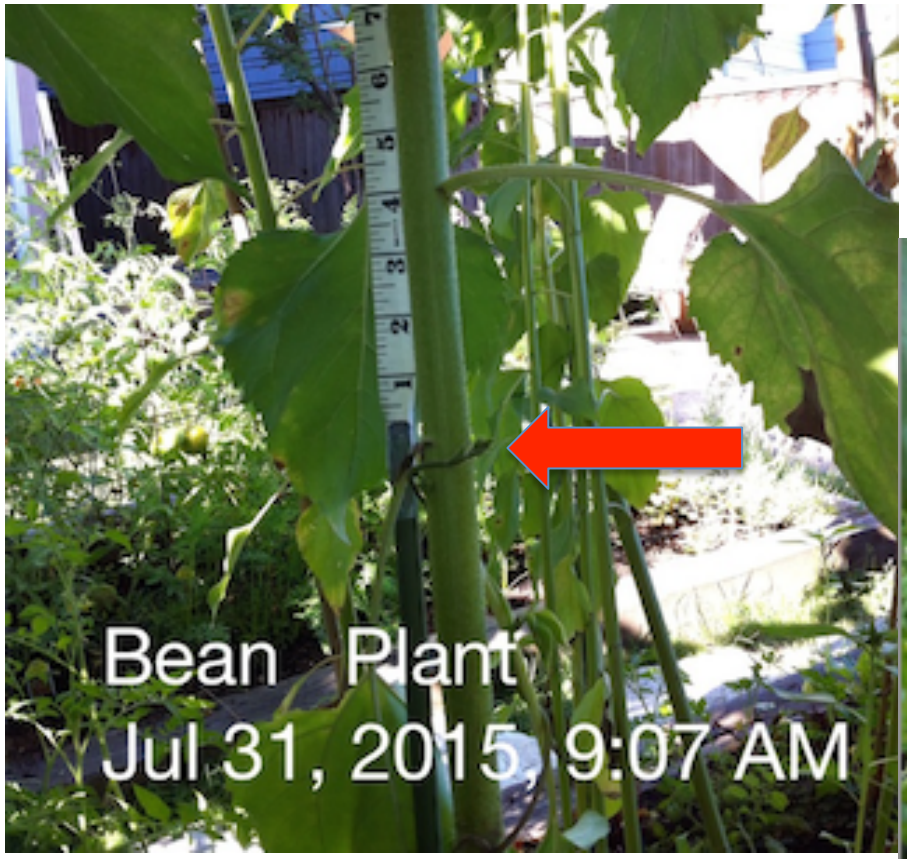
$$\boxed{\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}}$$

Growing vine

A bean grows up a pole in the form of a helix. If the vine tip grows at a constant rate α (cm/day) at what rate is its height above the ground changing?



Bean plant

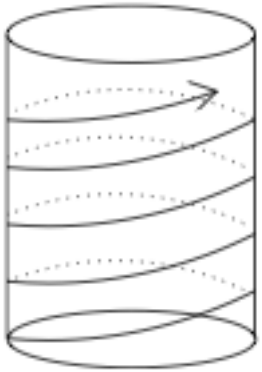


Growing vine

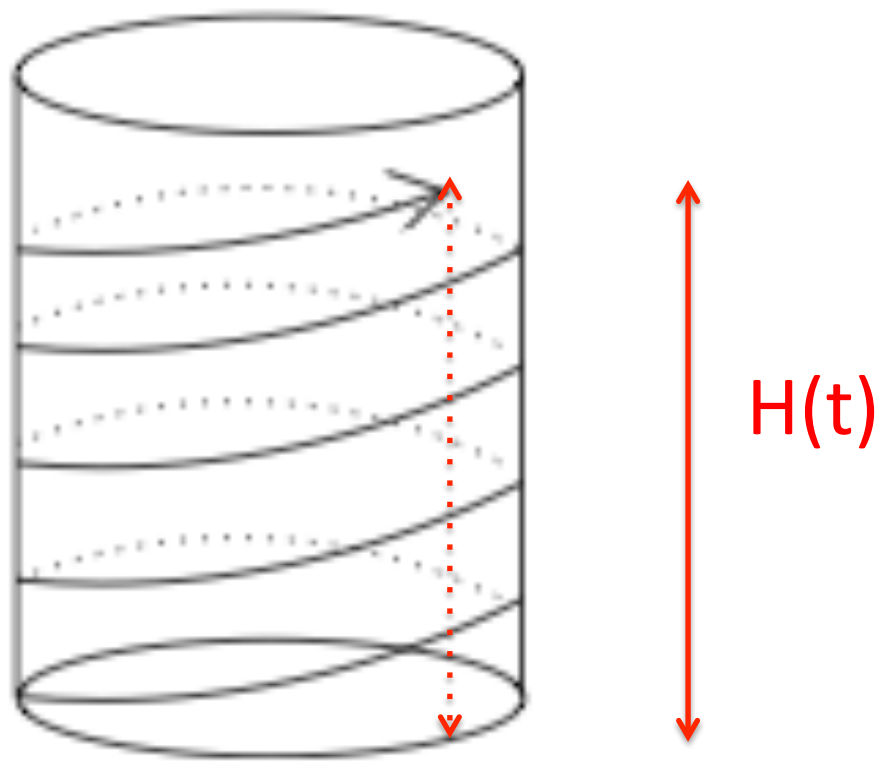
A bean grows up a pole in the form of a helix. If the vine tip grows at a constant rate α (cm/day) at what rate is its height above the ground changing?

Assume that the radius of the pole is r and the pitch of the helix is p, (p>0).

(pitch= height increase for each comple of the helix)

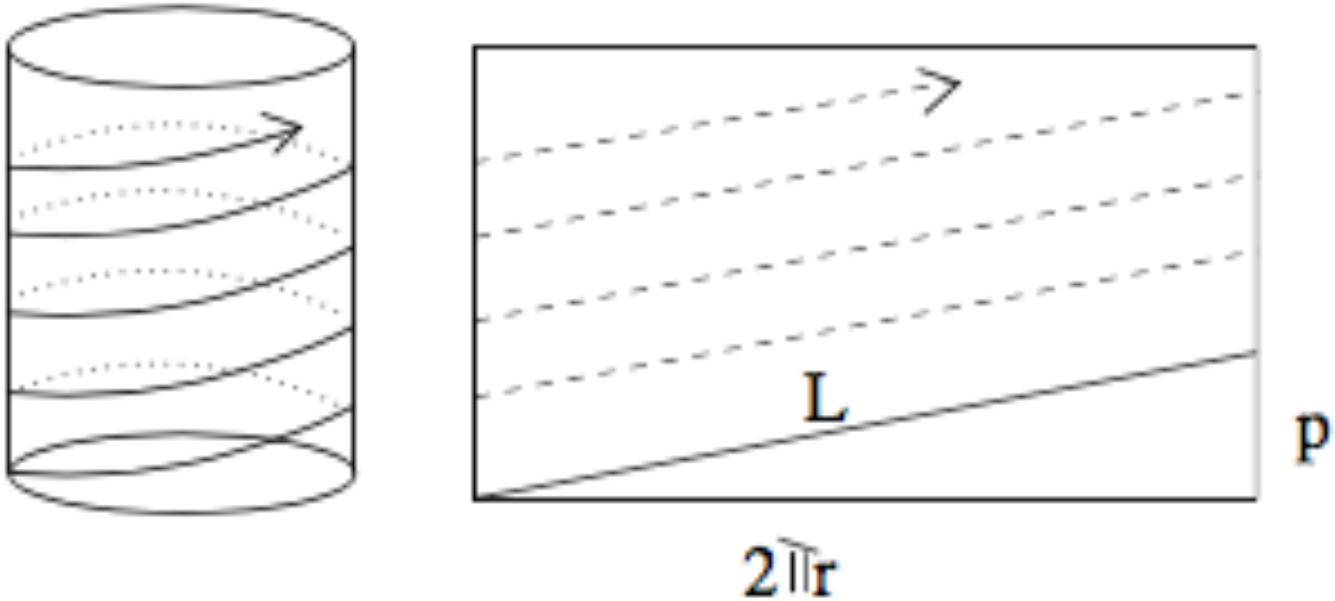


The geometry

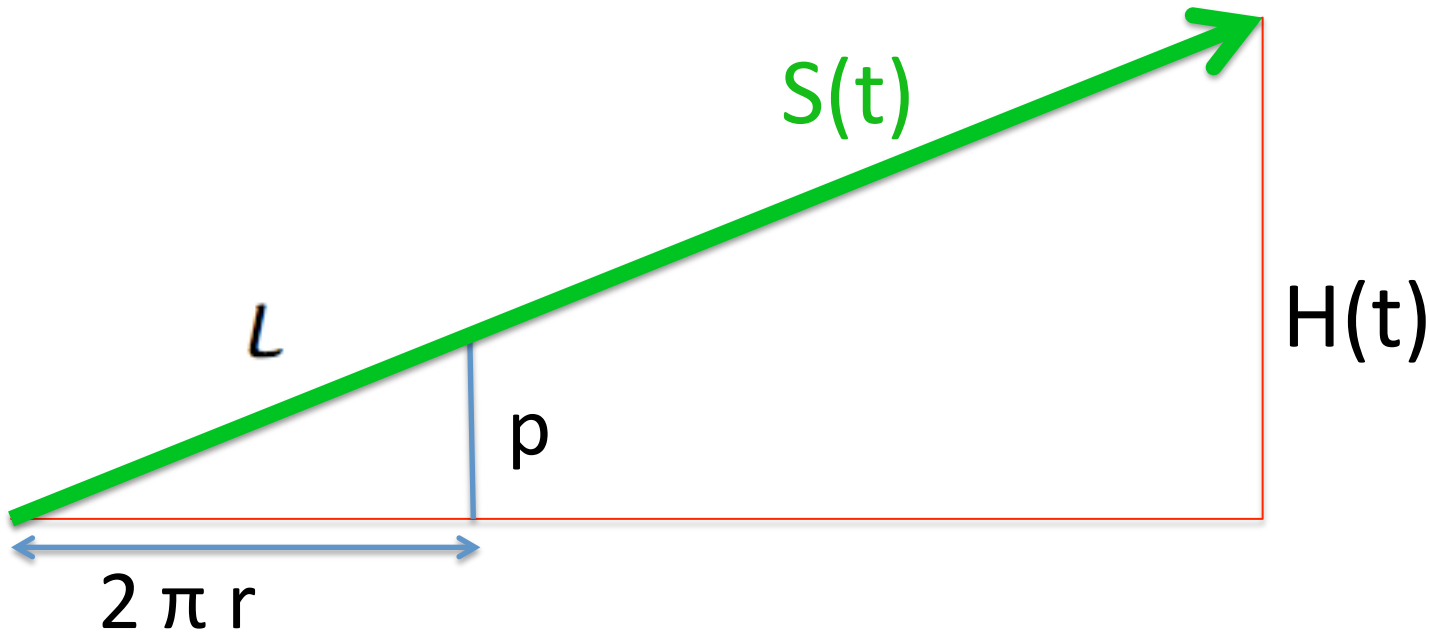


Unwrap it

- Unroll the cylinder and look at it:

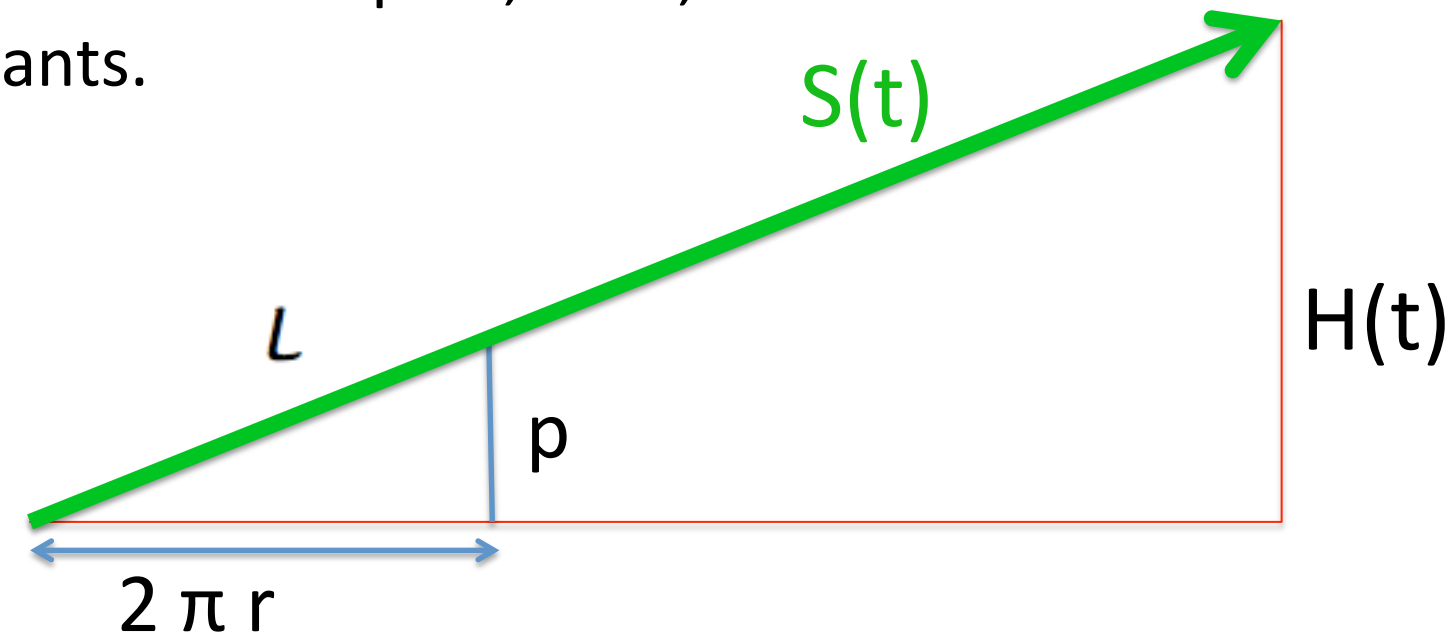


Unwrap the whole helix:



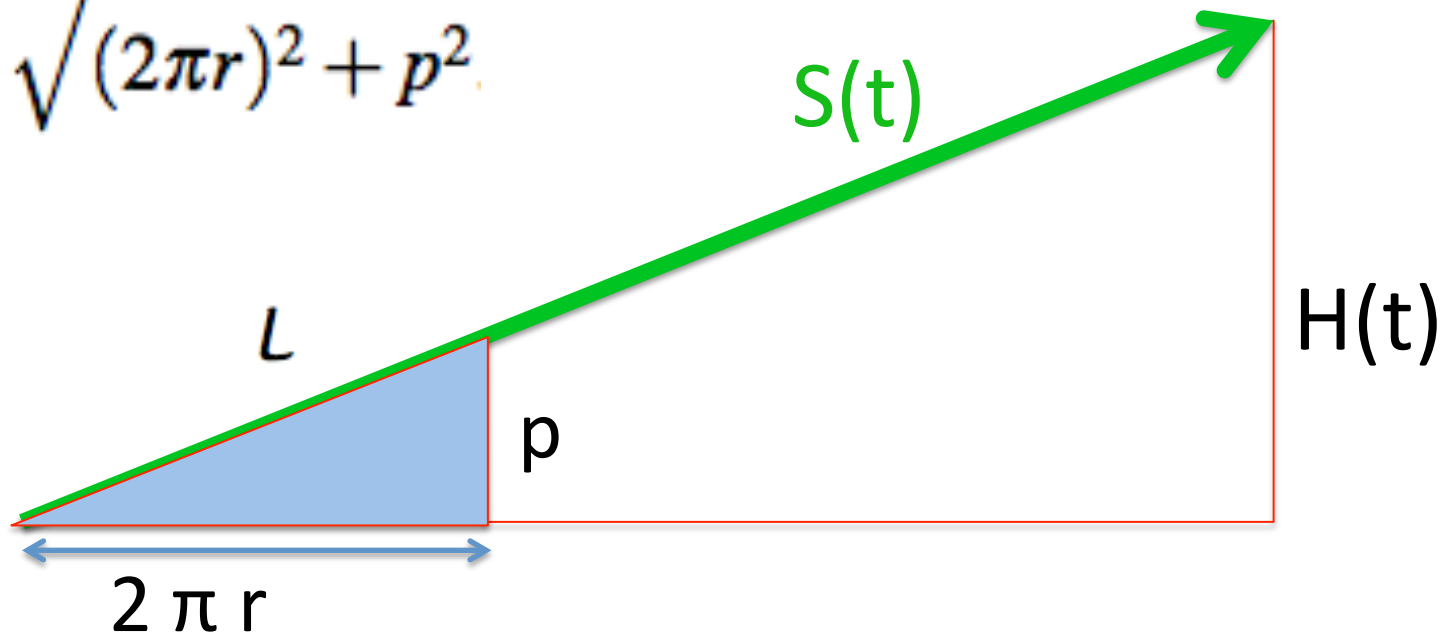
Unwrap the helix:

The pitch of the helix, p, and the circumference of the pole, 2 π r, are constants.



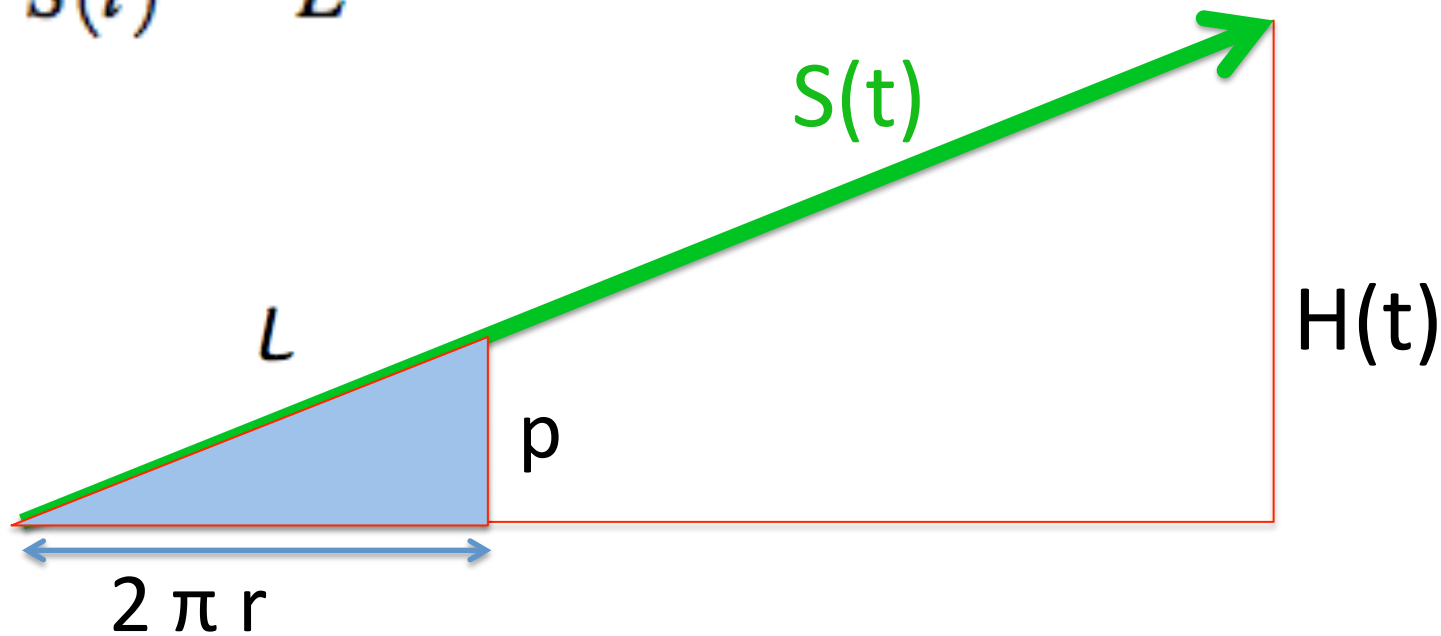
In the small triangle:

$$L = \sqrt{(2\pi r)^2 + p^2}$$



Similar triangles:

$$\frac{H(t)}{S(t)} = \frac{p}{L}$$



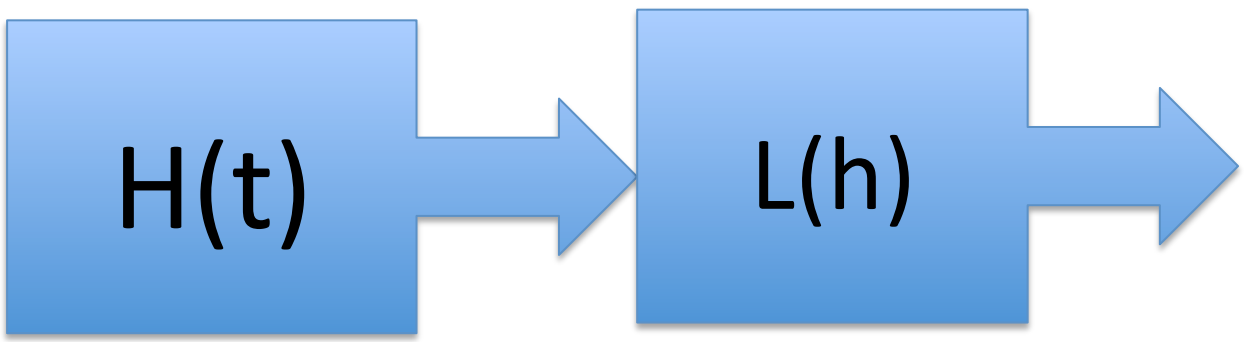
Solution – step 1: find the relationship

$$\frac{H(t)}{S(t)} = \frac{p}{L} = \frac{p}{\sqrt{(2\pi r)^2 + p^2}}$$

$$H(t) = S(t)\frac{p}{\sqrt{(2\pi r)^2 + p^2}}$$

Chain Rule

L(t) is related to h(t) which (we are told) increases at a constant rate.



Time t →

$$\boxed{\frac{dL}{dt} = \frac{dL}{dh} \cdot \frac{dh}{dt}}$$

Step 2- relate the rates

- Use the fact that p and r are constants

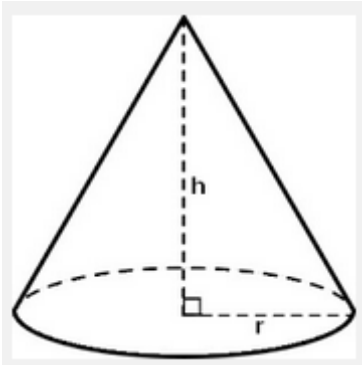
$$H'(t) = S'(t)\frac{p}{\sqrt{(2\pi r)^2 + p^2}}$$

- Given: Vine grows at constant rate, α:

$$dS/dt = \alpha$$

- Hence: $$H'(t) = \frac{\alpha p}{\sqrt{(2\pi r)^2 + p^2}}$$

Conical pile of sugar



Sugar is poured at a constant flow rate, 1 cm^3/s to form a conical pile. If the ratio of the height to radius of the cone is constant, at what rate is the radius of the base of the conical pile increasing? At what rate is the base area increasing?

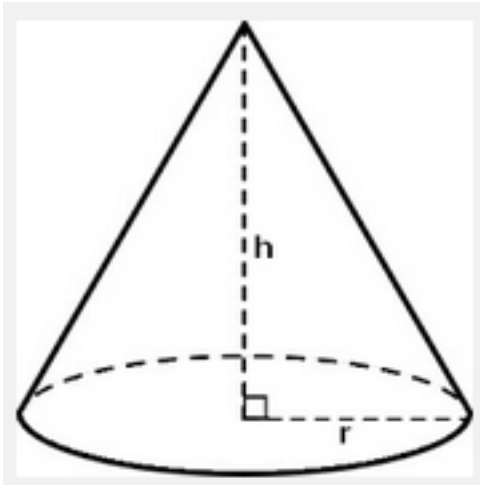
Note: the volume of a cone is $V = \frac{1}{3}\pi r^2 h$

The conical pile

The conical pile

The conical pile

Chain Rule

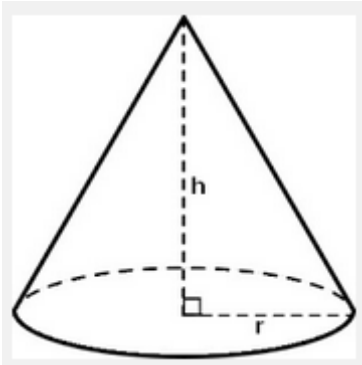


Time t →

$$r(t), h(t) \longrightarrow V(r,h) \longrightarrow$$

Use the fact that $h(t) = C\, r(t)$

Spend a few minutes trying this



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Note: the volume of a cone is $V = \frac{1}{3}\pi r^2 h$

Solution – step 1

Given:

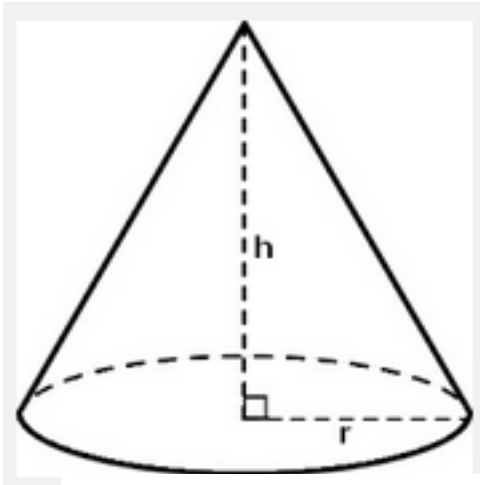
- Volume of cone $V = \frac{1}{3}\pi r^2 h$
- Ratio of height to radius is constant:

$$\frac{h(t)}{r(t)} = \text{constant} = C$$

Hence:

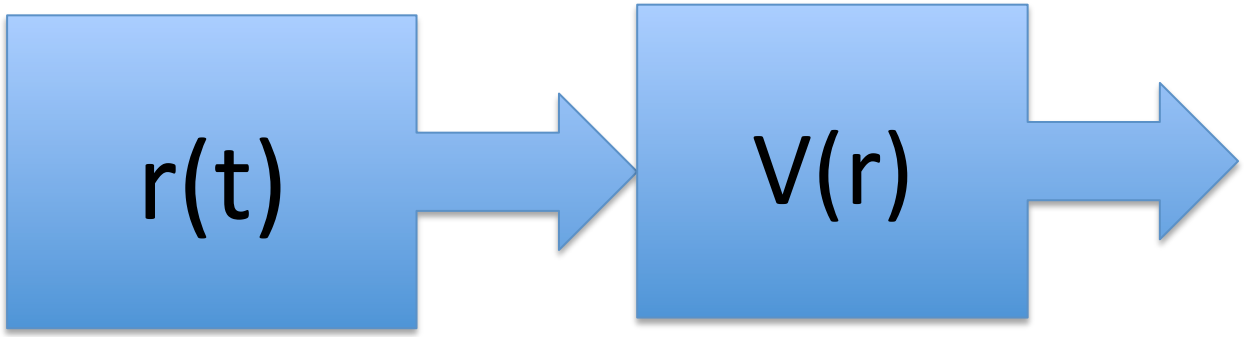
$$V = \frac{1}{3}\pi r^3 \cdot C$$

Chain Rule



$h(t) = C\, r(t)$

Time t →



$$\boxed{\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}}$$

Solution – step 2

$$V = \frac{1}{3}\pi r^3 \cdot C$$

$$\frac{dV}{dt} = \frac{1}{\cancel{3}} C \pi \cdot \cancel{3} \cdot r^2 \frac{dr}{dt}$$

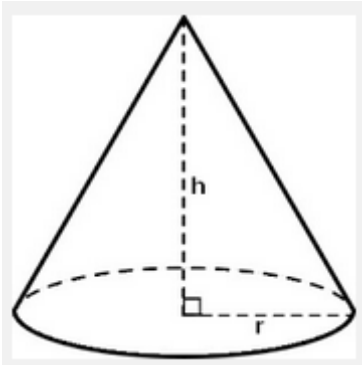
$$= C \pi r^2 \frac{dr}{dt}$$

$$1 \frac{cm^3}{s} = C \pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{C \pi r^2} \quad cm/s$$

Problems to test your skill

Related problem:



Try this one yourself!

Sugar is poured at a constant flow rate, 1 cm^3/s to form a conical pile. At what rate is the base area increasing?

$$V = \frac{1}{3}\pi r^2 h$$

Related test problem

3. **[3 pt]** Oil is leaking out of a cargo ship at the rate of 1 m^3/hr, forming a circular patch on the surface of the water (an "oil slick"). The radius $r(t)$ of the oil slick increases while its thickness, $\tau = 0.01$ m, is constant. Find the rate of change of the radius, at the moment when $r = 10$ m. Reminder: The volume of a cylinder is $V = \pi r^2 h$.

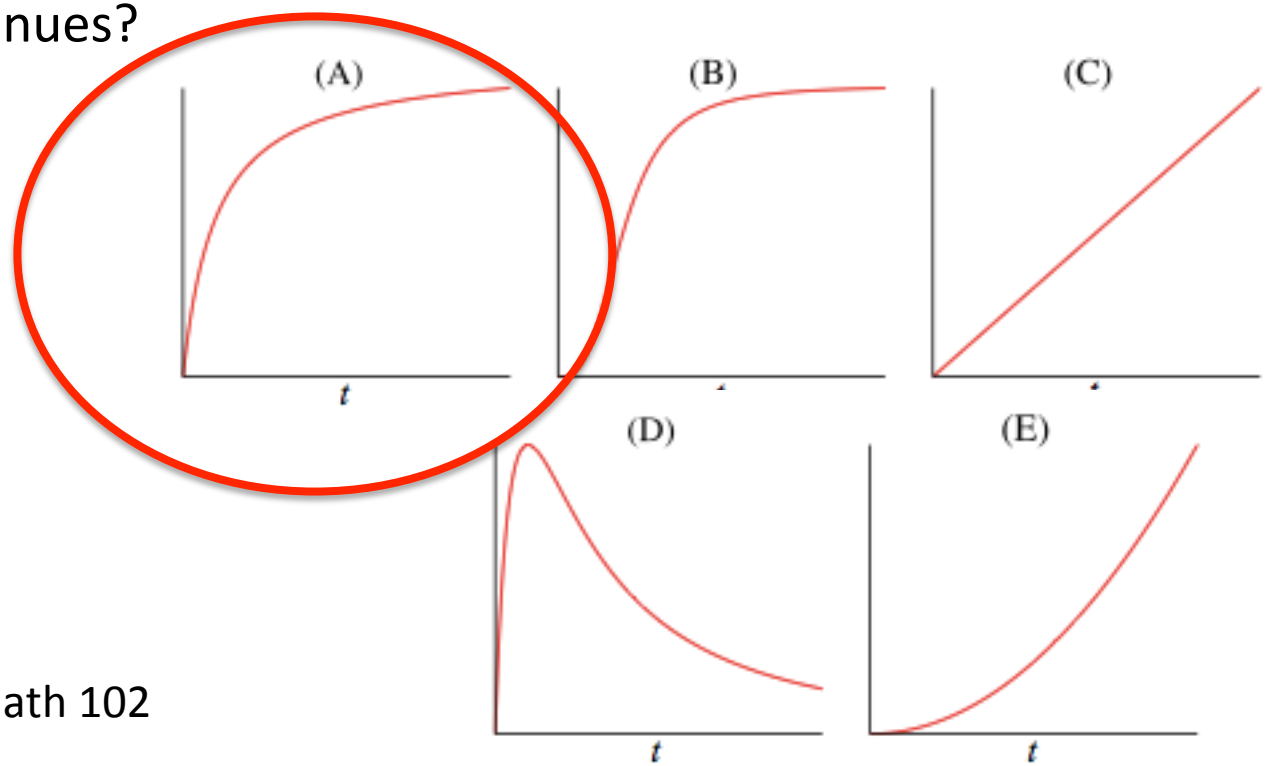
•

Solutions to previous problems

First try out the problems at the end of the last lecture slides. Only then should you “peek” at the answers.

MC 3

Each of the graphs represents f(t), the amount of food gained during time t spent in a food patch. Which of these graphs represents the type of patch where food can be collected rapidly at first, but there is only some maximal amount that can be obtained no matter how long the search continues?



MC 5

The absolute maximum of the function

$y = f(x) = x + 1/x$ on the interval $0.1 \le x \le 2$

occurs at

(a) $x = -1$

(b) $x = 0.1$

(c) $x = 1$

(d) $x = 2$

(e) $x = \pm 1$

MC 6

Let y = f(x) be a smooth function (derivatives of all orders exist) at x0. Which of the following statements is correct?

(a) If $f''(x_0) = 0$, then the function has an inflection point at x_0.

(b) If the function has an inflection point at x_0, then $f''(x_0) = 0$.

(c) Both (A) and (B) are correct.

(d) If $f''(x_0) = 0$, then the function has a critical point at x_0.

(e) If $f''(x_0) = 0$, then the function never has a critical point at x_0.