The derivative

Analytic, geometric, computational

UBC Math 102

Derivative: Analytic

Use the definition of the derivative to compute the derivative of the function $y = x^3$

MT test-type question

Done fast? Try to extend your method to $y = x^n$
Where n is any positive integer.

More advanced

Sols: see last slides

Math Learning Center

- The MLC is a space for undergraduate students to study math together, with friendly support from tutors (grad students in math).
- Located at LSK310 and LSK302
- Hours: open 5 days a week.
- Note: MLC is not a place to check HW answers or receive solutions. MLC aim is to aid students in becoming better learners and to develop critical thinking skills.
- http://www.math.ubc.ca/~MLC/.

Diagnostic test results

Final Grade in M102 ≈ DT score + 10%

If your DT < 40% → high risk of failing

Possible action: take M180 instead of M102

OR : prepare to work extra- hard

Help with homework

Assignment2: Problem 16

- Evaluate the limit, if it exists. If a limit *does not exist,* enter DNE.

$$\lim_{x \to 7} \frac{x^2 - 5x - 14}{x^2 - 4x - 21}$$

- Hint: consider factoring top and bottom

Assignment2: Problem 18

- Evaluate the limit by "guessing" $L = \lim_{x \to 0} \frac{e^x - 1}{x}$

- We have not learned enough about this function to actually compute the limit, but we can use a value close to x=0 to guess the limit

“ Guessing”

To “guess” a value for this limit: $\lim_{x \to 0} \frac{\sin(x)}{x}$

I would input which of the following into WebWork?

(A) $\frac{\sin(0)}{0}$ (B) $\frac{\sin(1)}{1}$ (C) $\frac{\sin(\pi)}{\pi}$ (D) $\frac{\sin(0.001)}{0.001}$

(E) None of the above

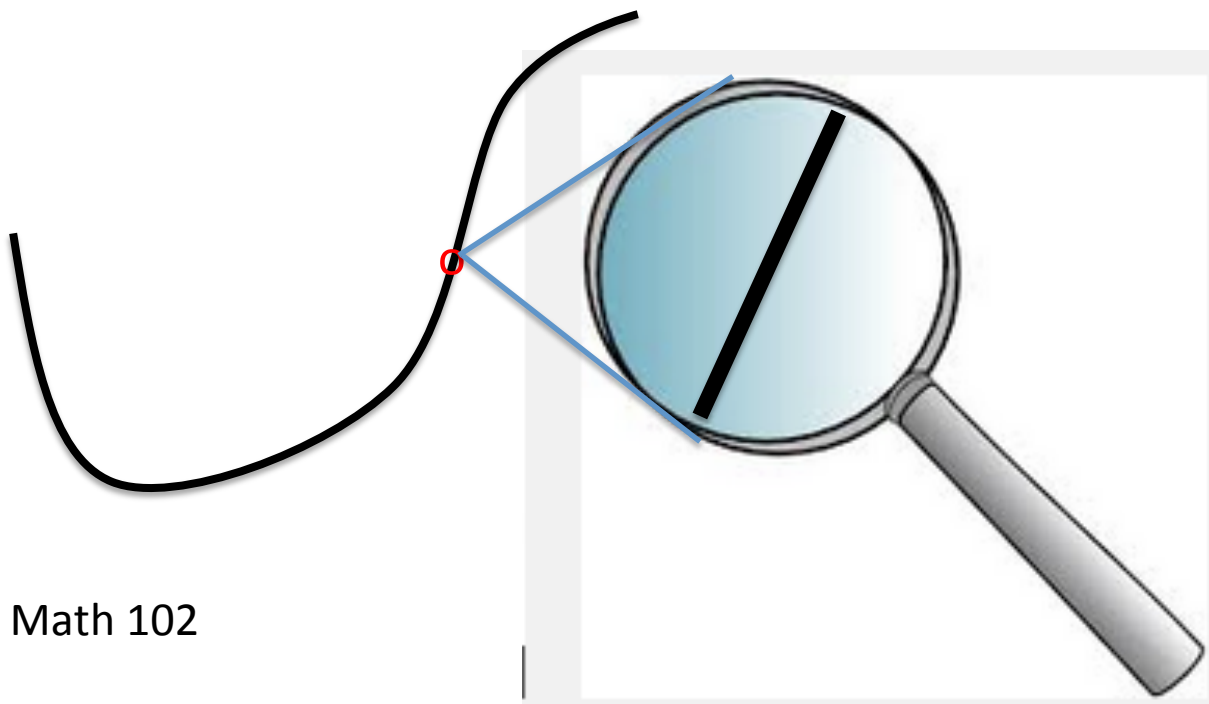
The derivative

Geometric

UBC Math 102

Tangent line

- It is the line we see when we zoom into the graph of a function at some point

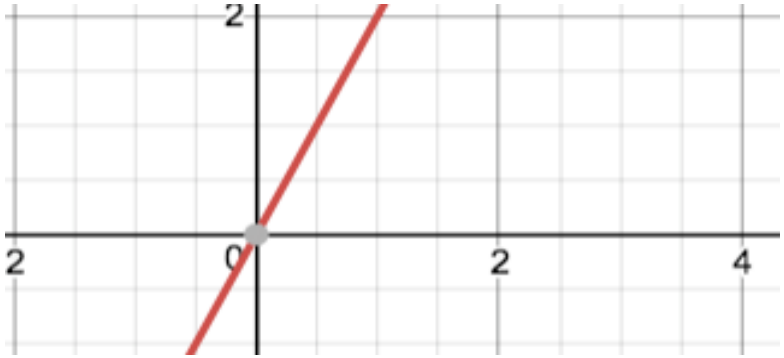


Simple cases:

What is the derivative of these functions?

$y = C$

$y = a\,x$



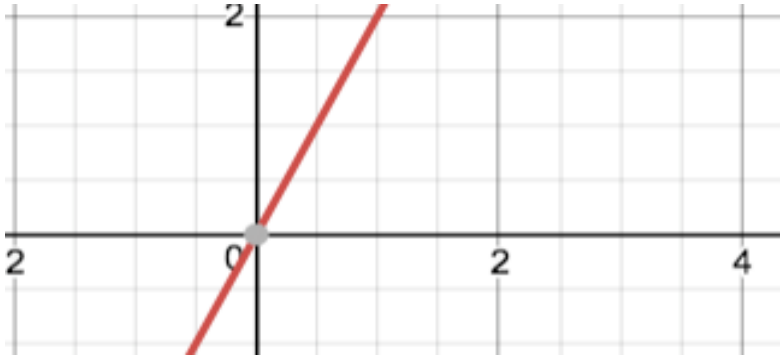
Simple cases:

The derivative is

$y = C$

$y' = 0$

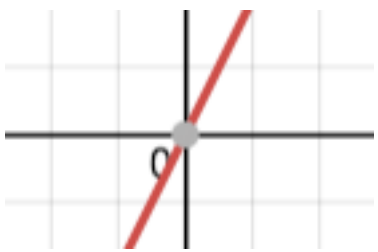
$y = a\,x$



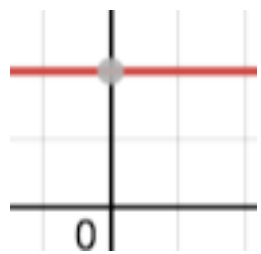
$y' = a$

Remembering slopes

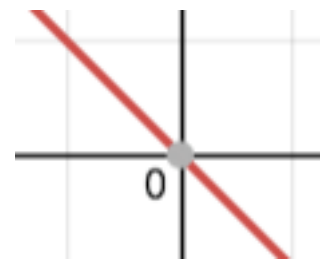
Positive



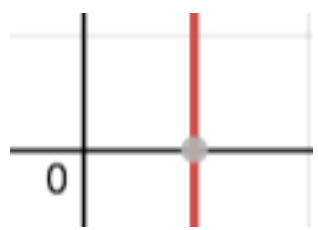
zero



negative

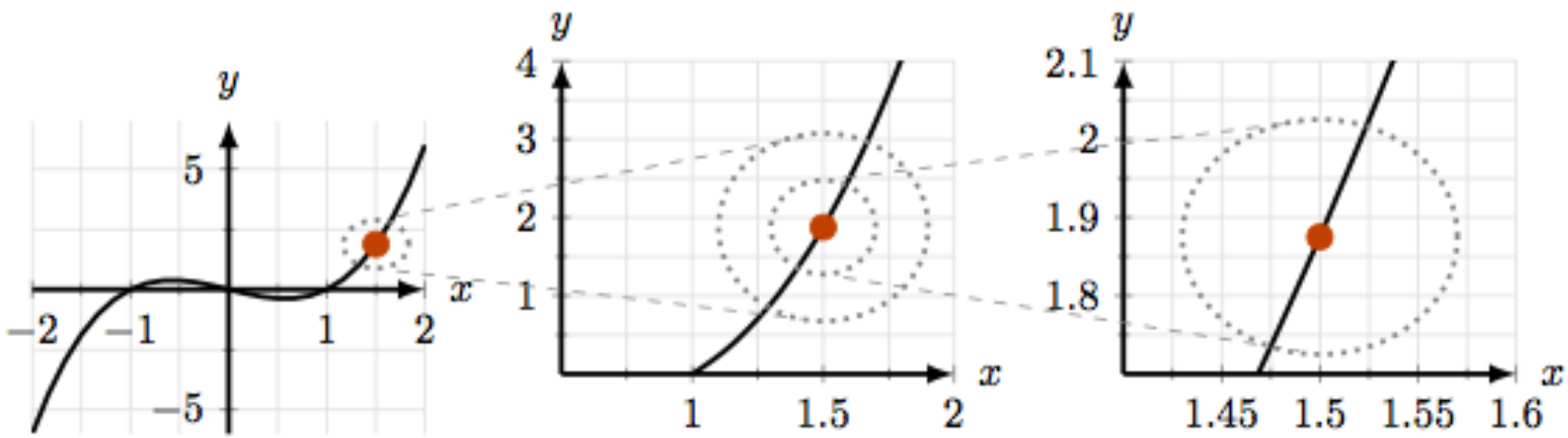


"infinite" (undefined)



Zooming into the graph of $y = f(x) = x^3 - x$

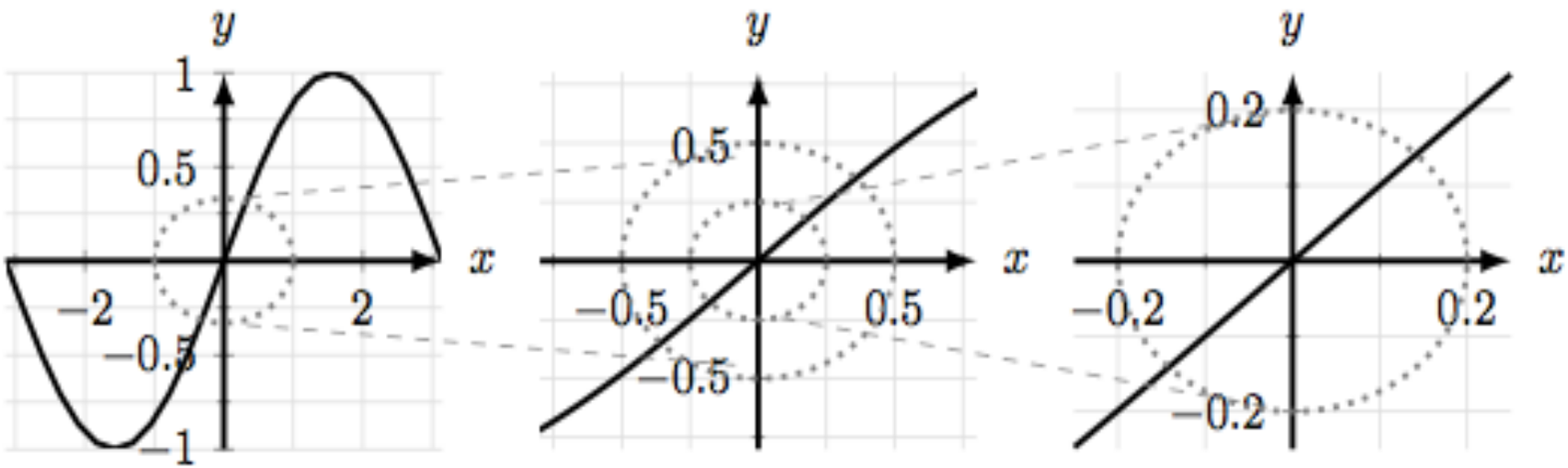
- At the point x=1.5



See course notes

Zooming into the graph of $y = f(x) = \sin(x)$

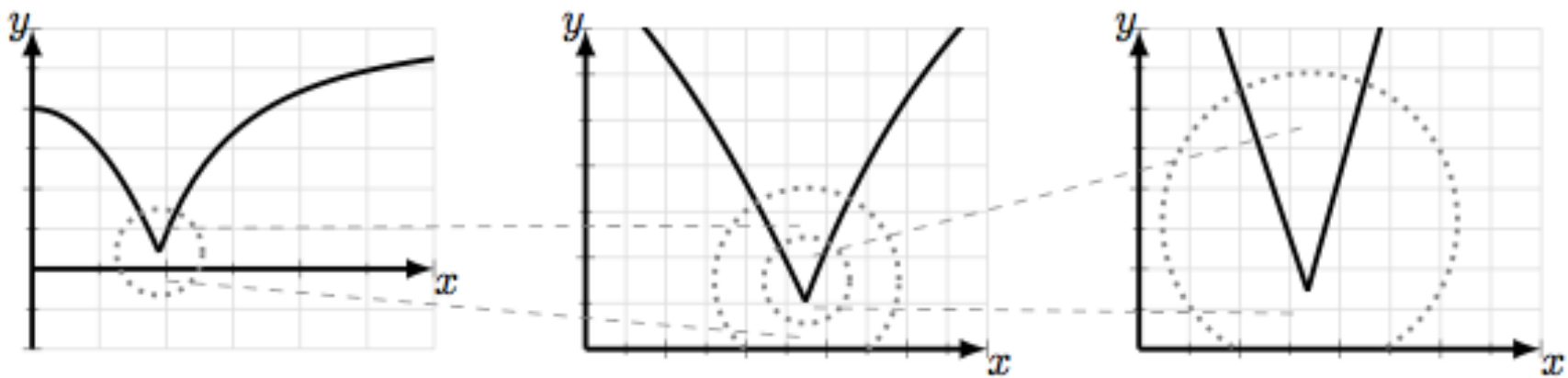
At x=0



See course notes

A function with a cusp

At the cusp, the derivative does not exist.



See course notes

When does the derivative exist?

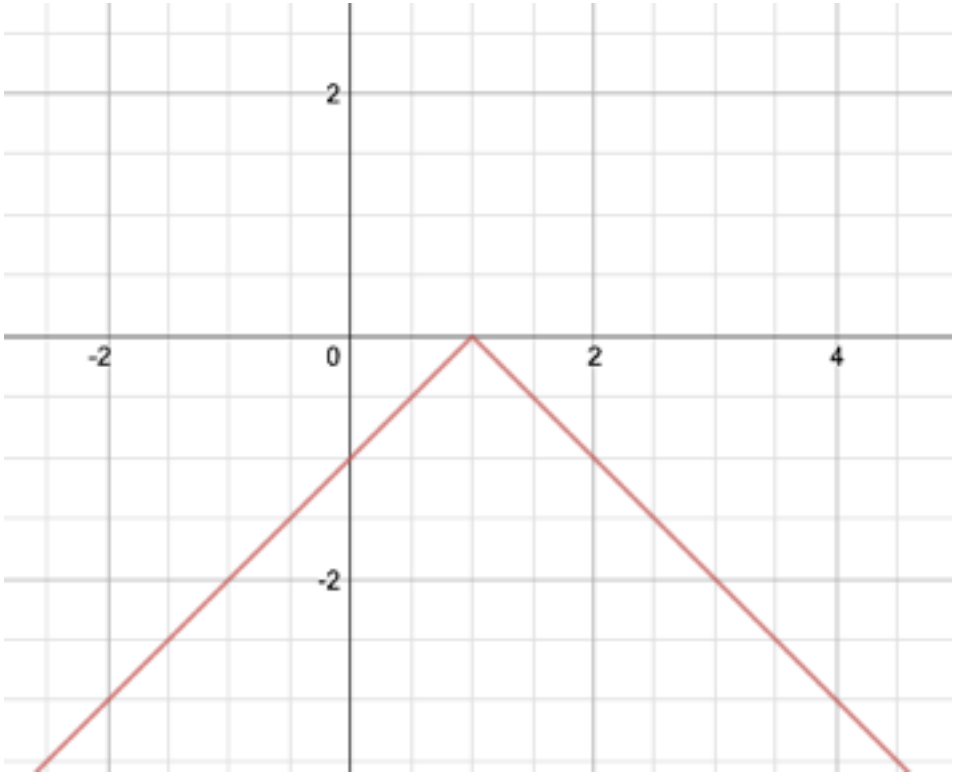
If a function is discontinuous at x=*a* then its derivative is not defined at that point.

If a function has a cusp at x=*a*, then its derivative does not exist (is not defined) at that point.

If a function "blows up" (goes to infty) at x=*a*, then its derivative does not exist at that point.

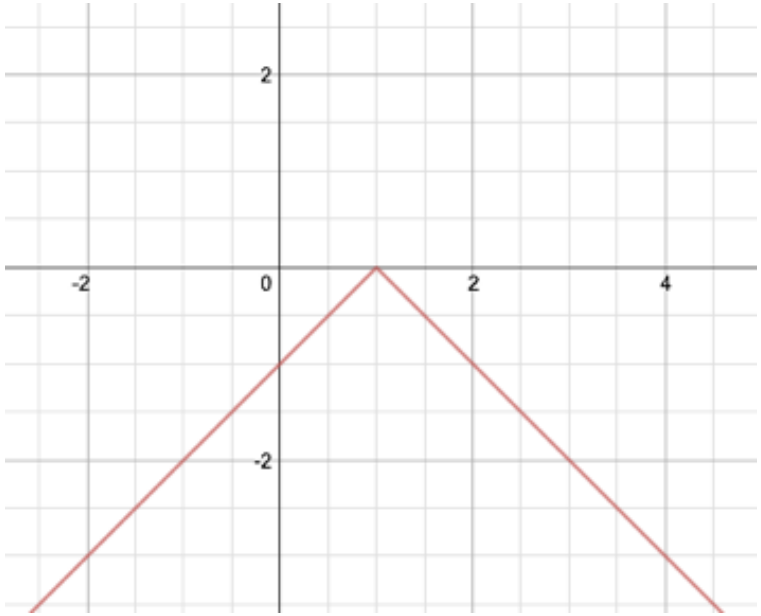
1. What is this function?

- A $|x+1|$
- B $|x-1|$
- C $-|x+1|$
- D $-|x-1|$
- E $-(x-1)$

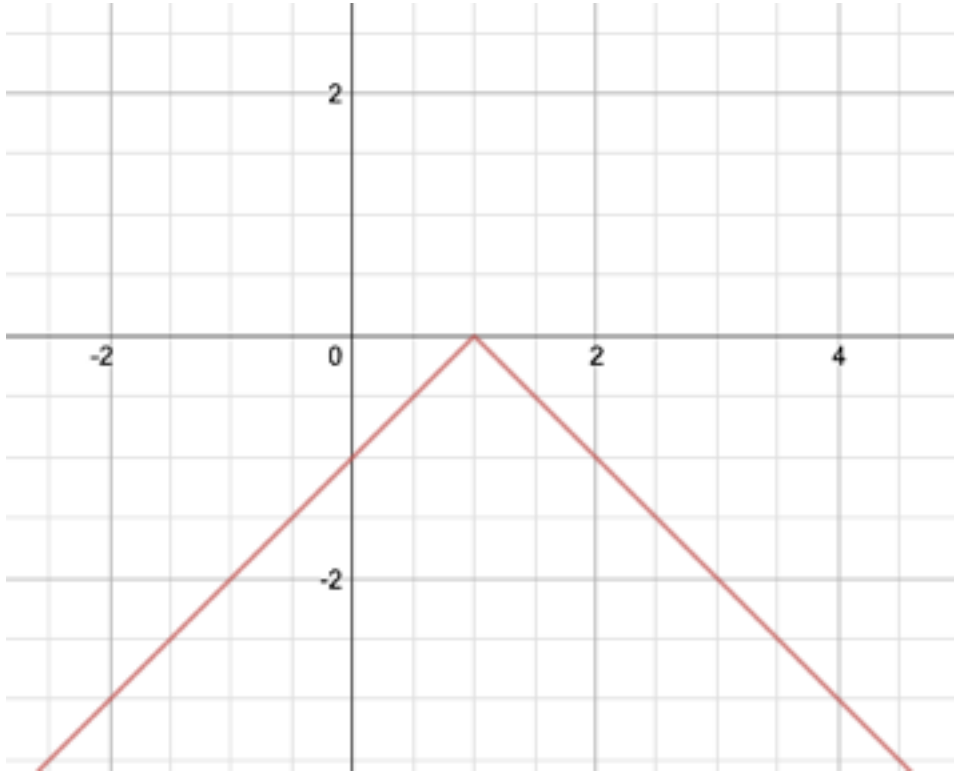


2. This function is

- A discontinuous
- B not differentiable anywhere
- C has no limit as x->0
- D differentiable except at x=1
- E I am confused

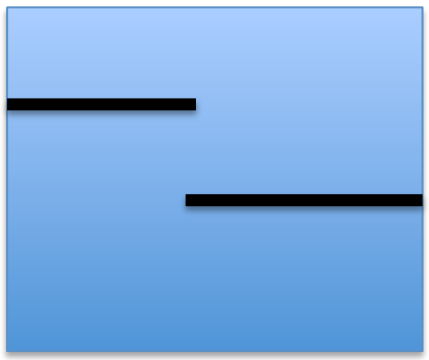


Sketch the derivative of this function

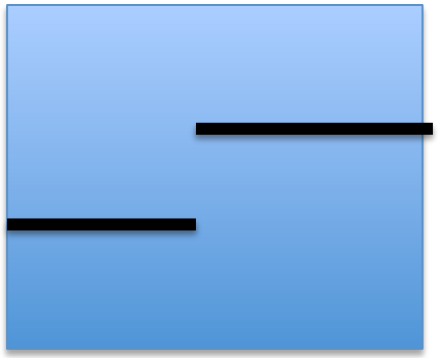


3. The derivative of this function looks like:

- A



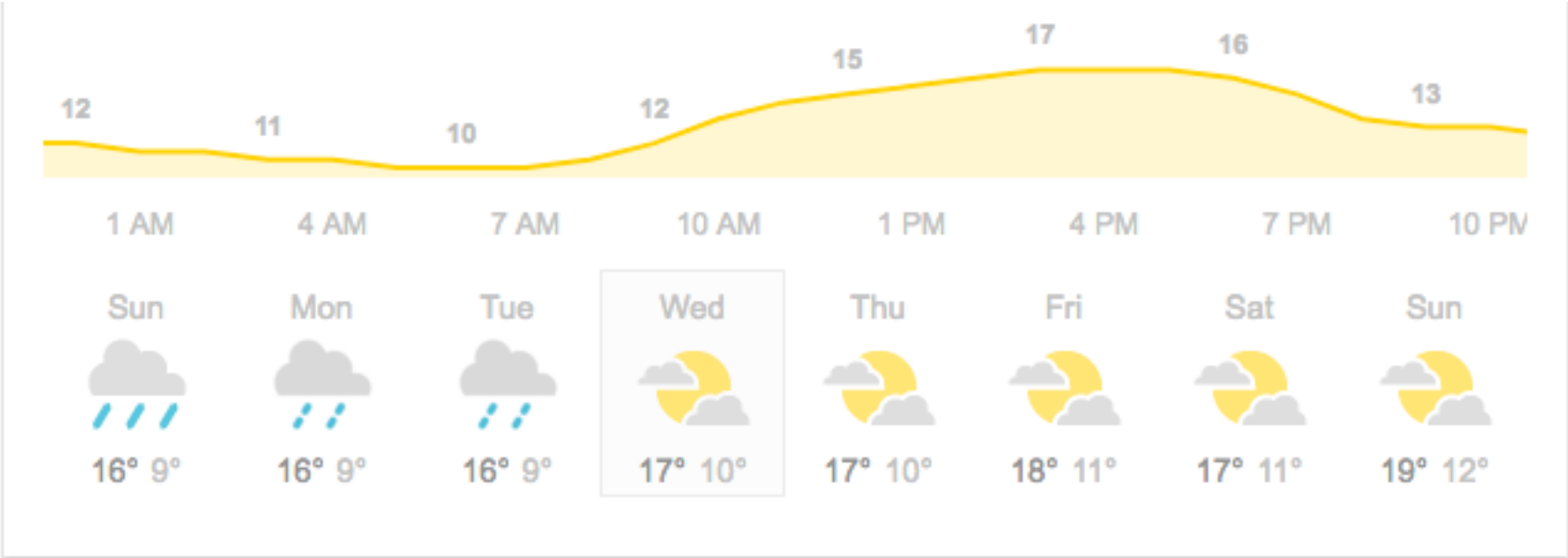
- B



Daily temprature

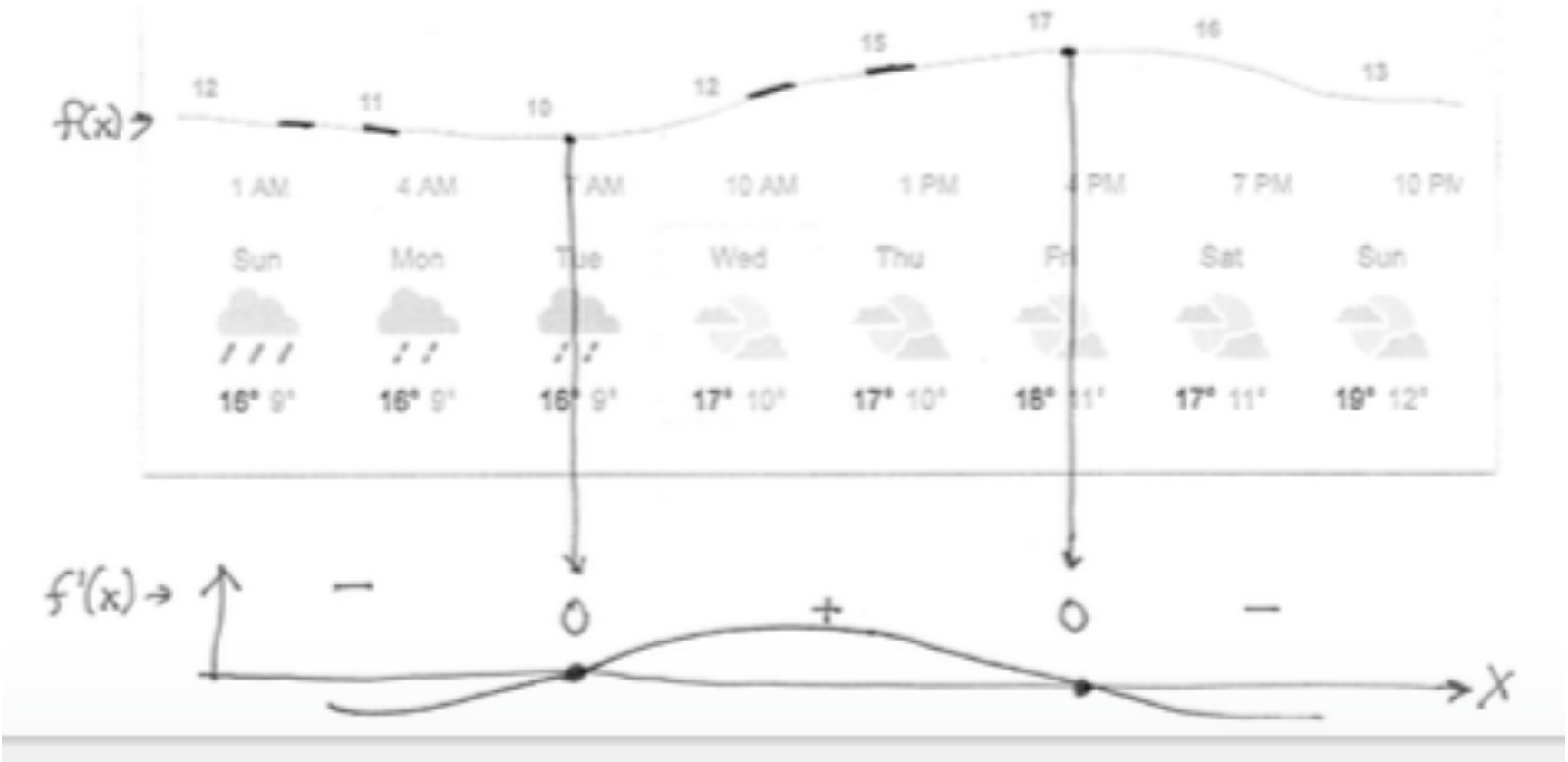
(Forecast for Wedn Sept 20)

Sketch the rate of change of temperature.



Solution in class

- The local max and min are places where the derivative is zero



Solar power

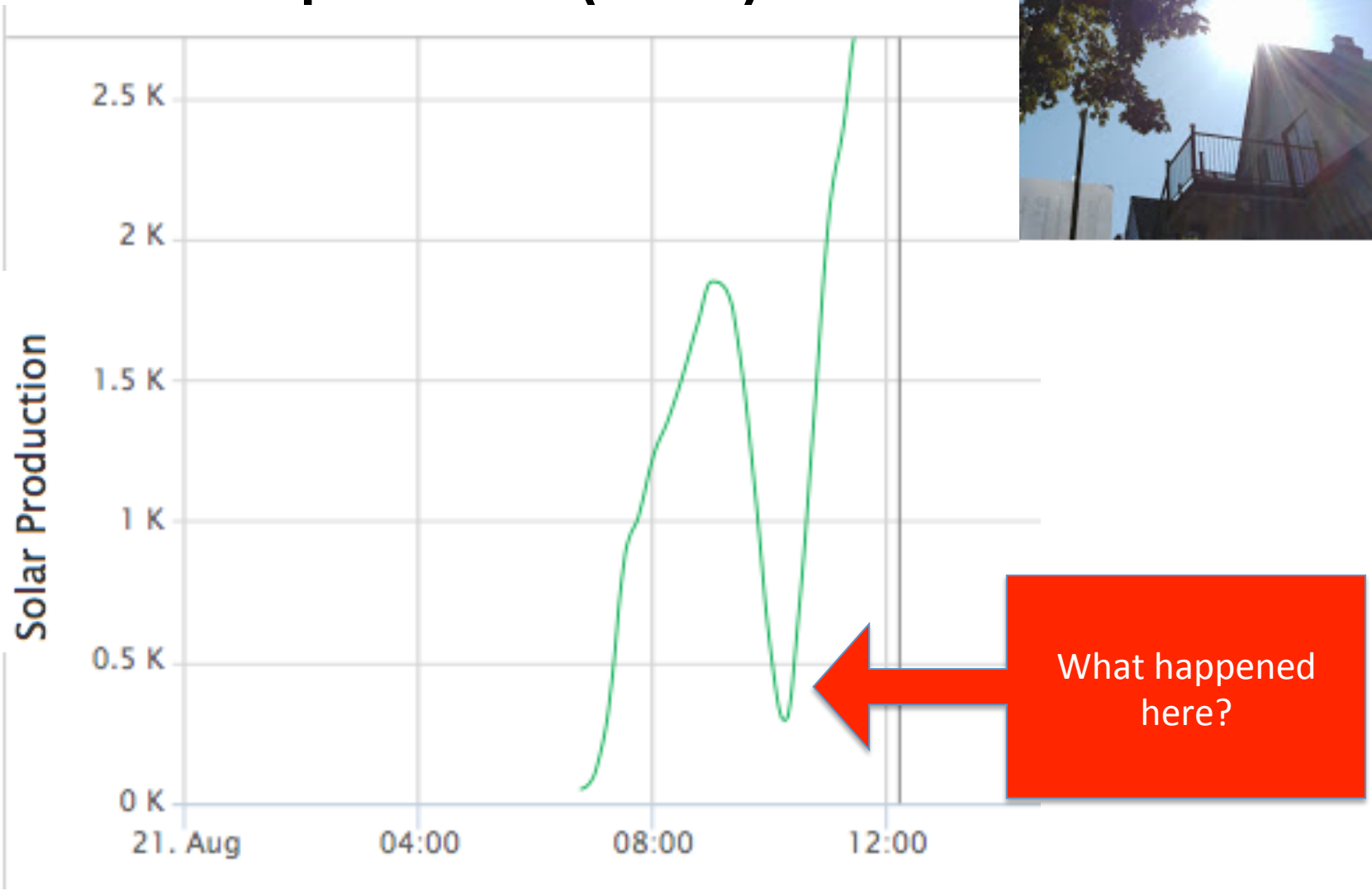
Vancouver Renewable Energy

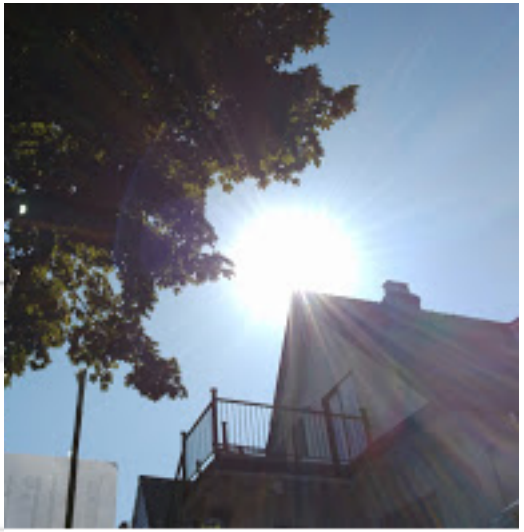
Rooftop solar collectors

Sustainability and clean energy

Current Power	Energy today	Energy this month	Lifetime energy	Lifetime revenue
2.87 kW	8.89 kWh	377.5 kWh	3.33 MWh	C$432.26

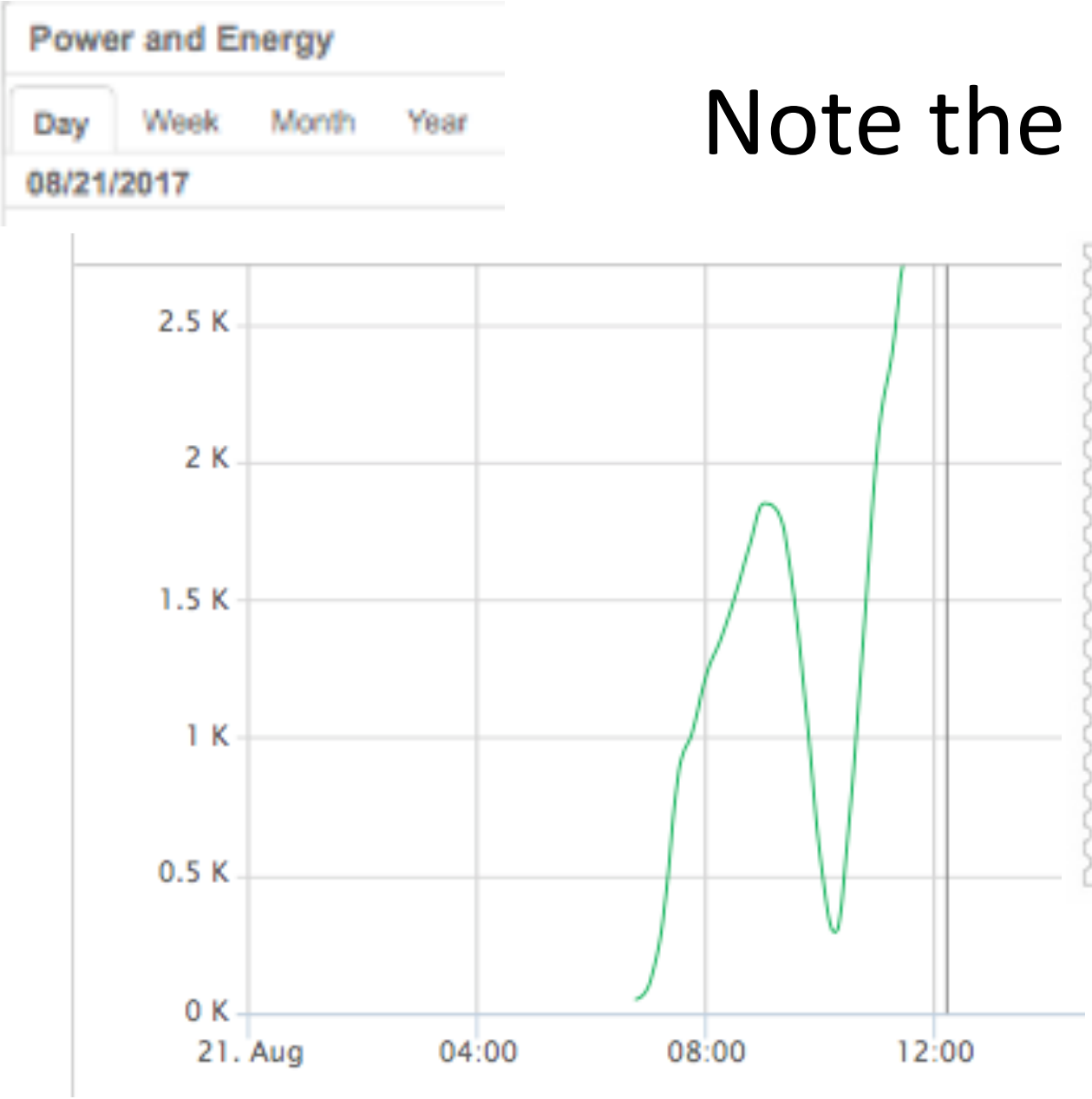
Solar power (KW)



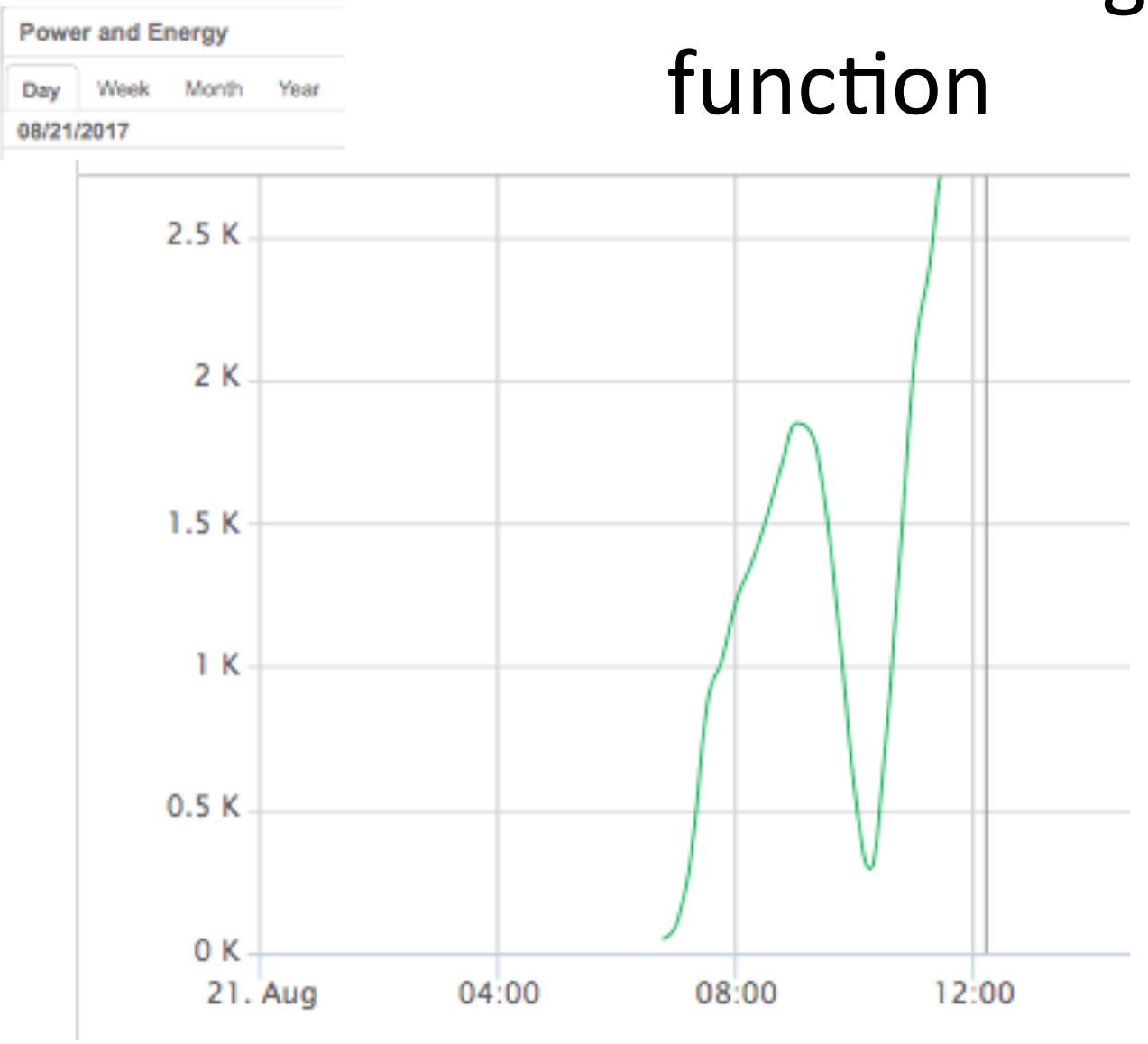


What happened here?

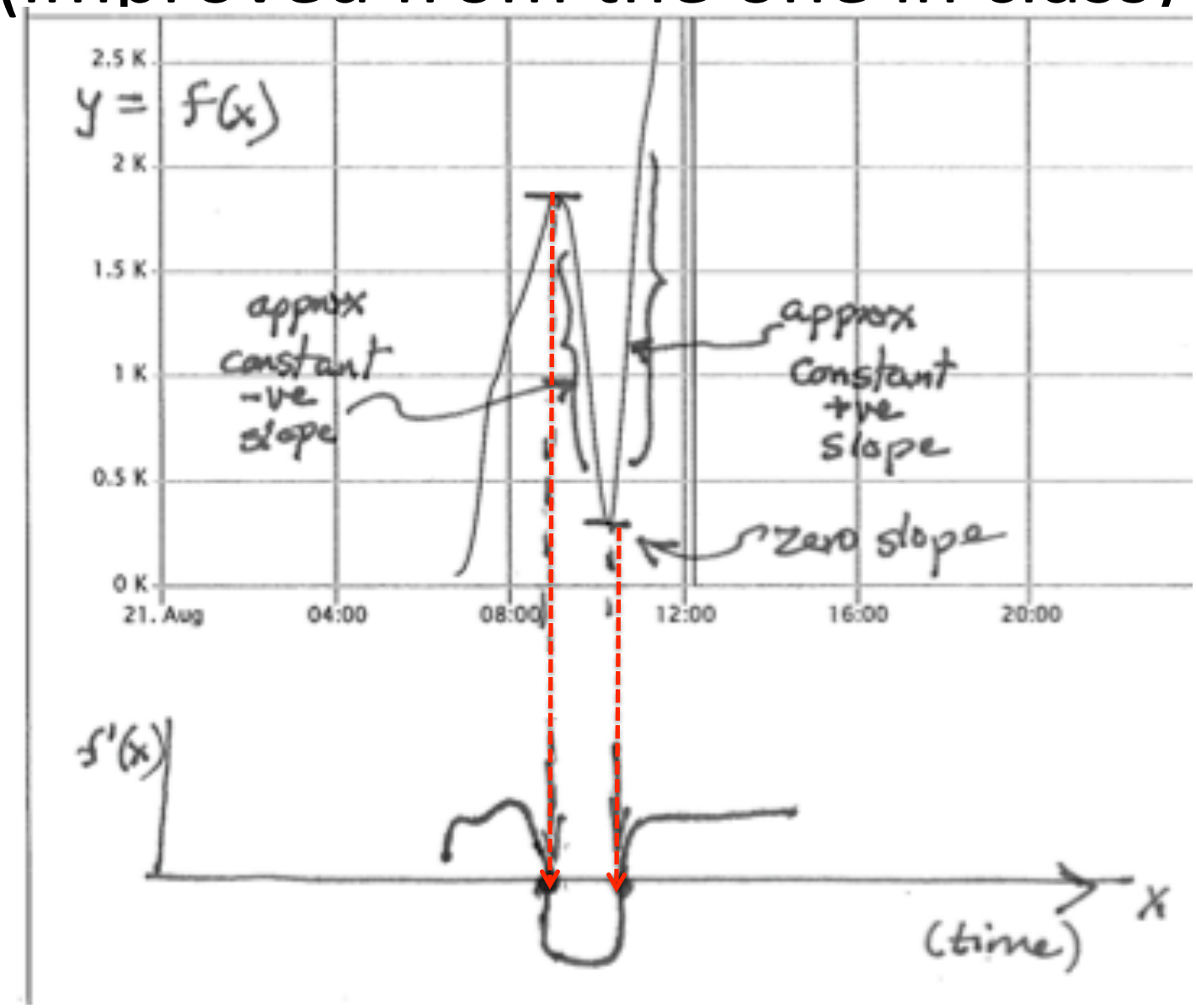
Note the date



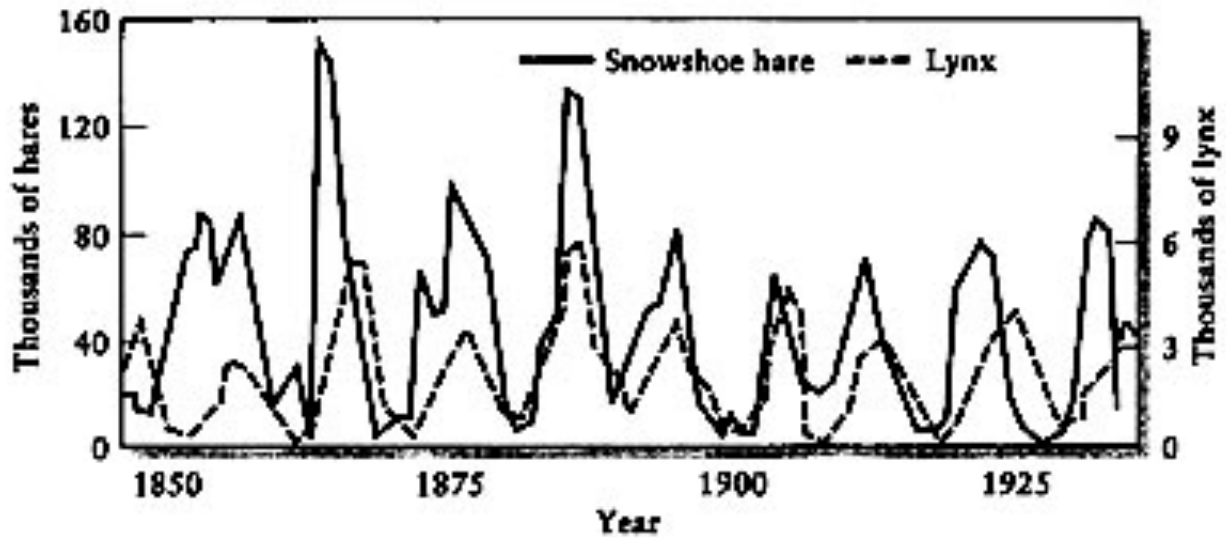
Sketch the rate of change of this function



Solution
(improved from the one in class)

- 

Predator-prey population cycles

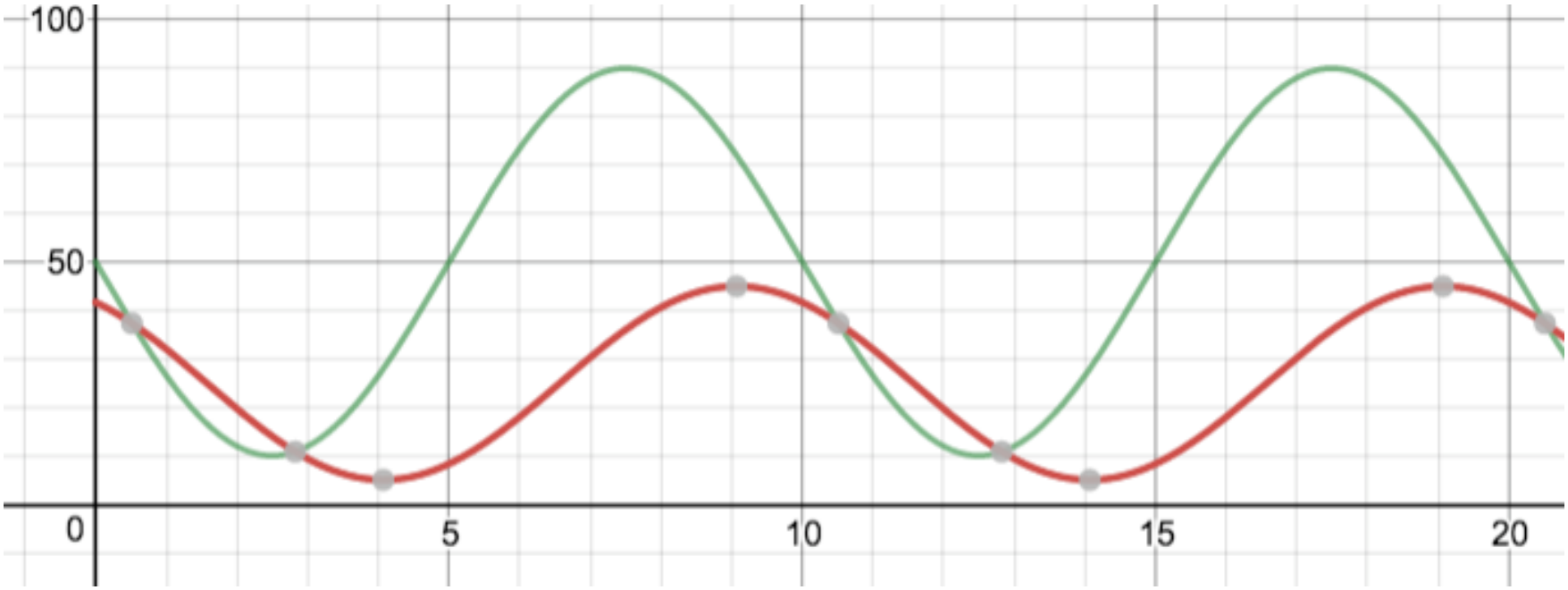


We have a great record of cycles of predators and prey (Lynx and Hare) in Canadian arctic

why??

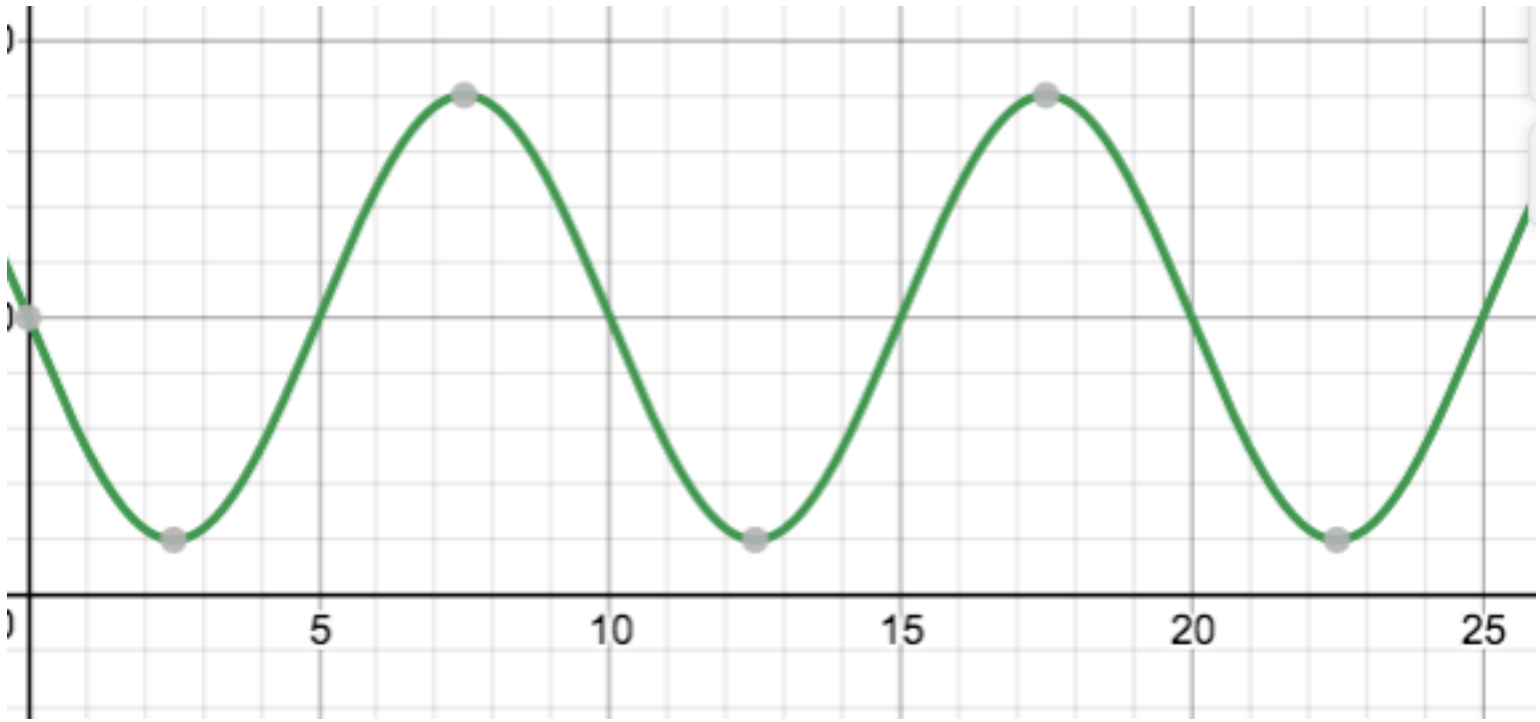
Overall trend

- Predator and Prey (in thousands)



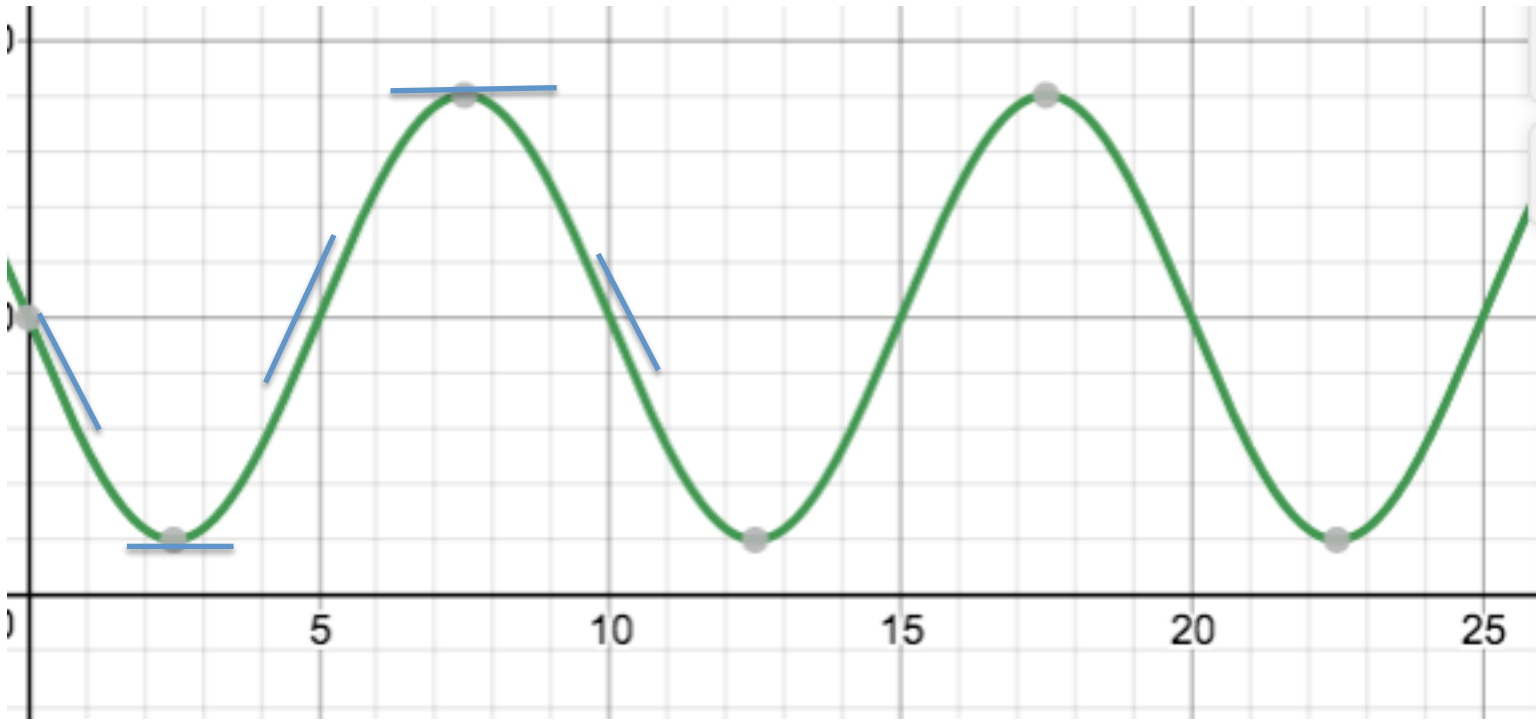
Just ther prey

Sketch the rate of change dP/dx of the prey population



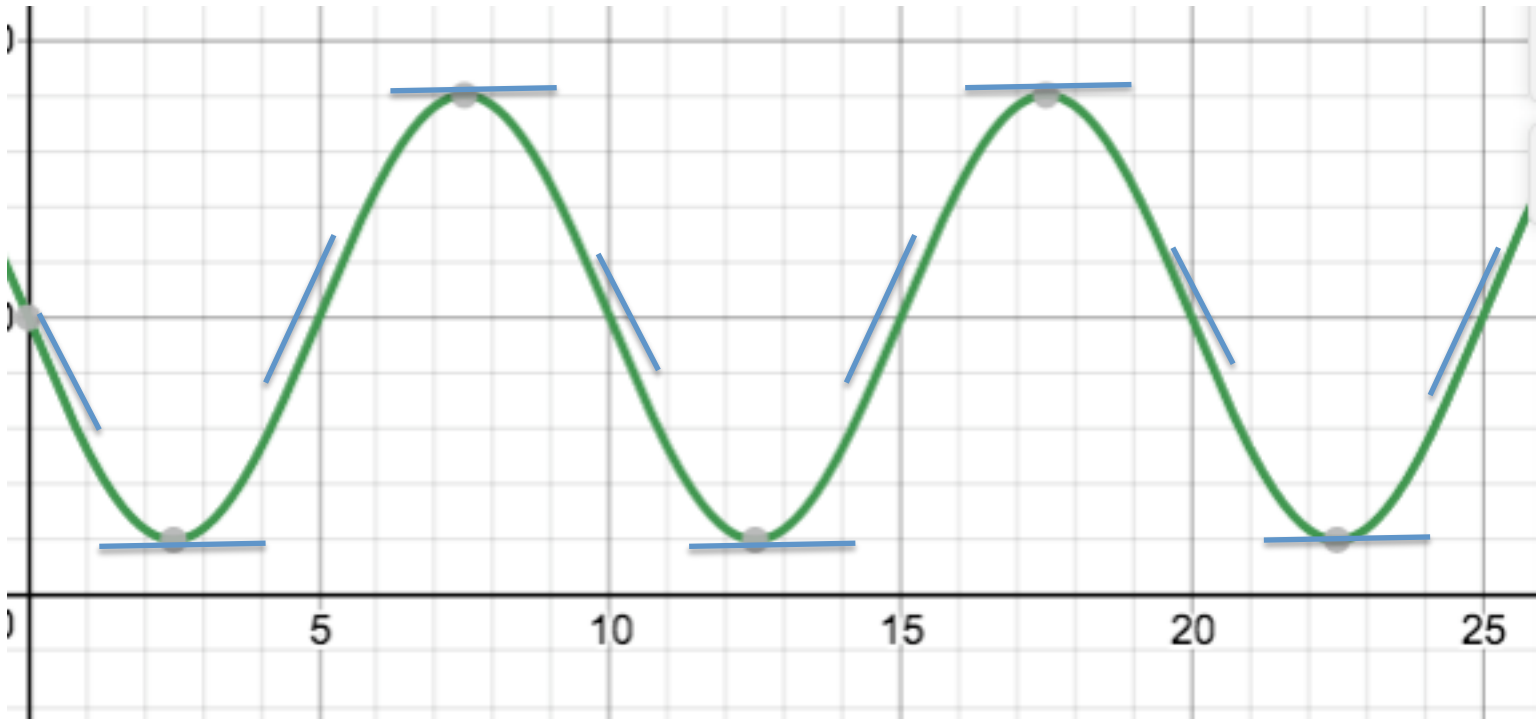
Just ther prey

Sketch the rate of change dP/dx of the prey population



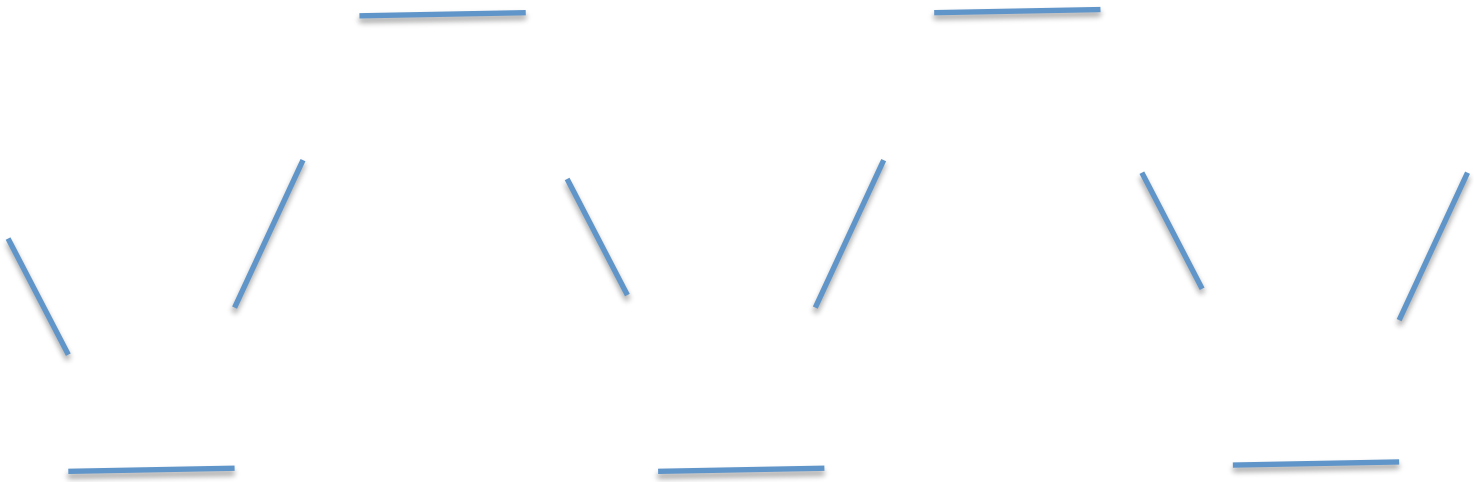
Just ther prey

Sketch the rate of change dP/dx of the prey population



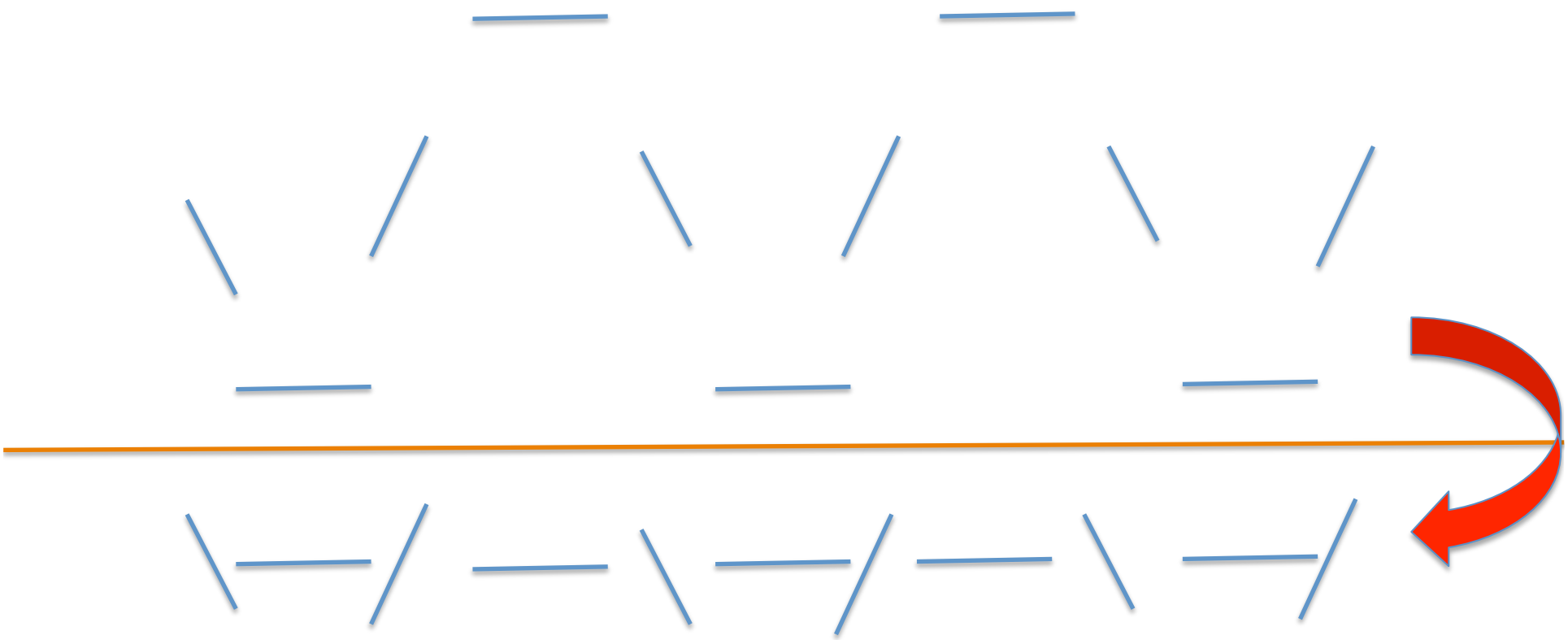
Tangent lines

Now we consider the slopes of the tangent lines



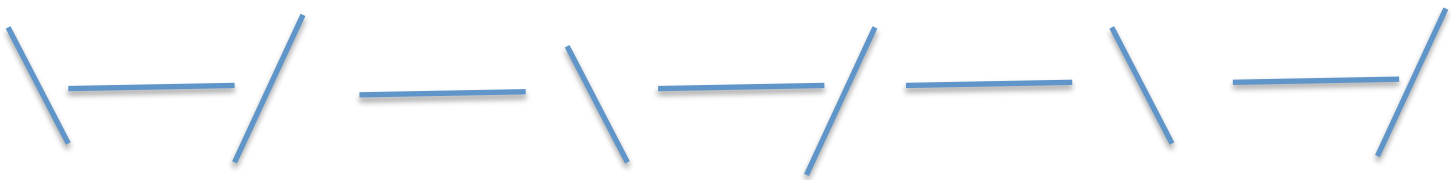
Tangent lines

Put them all along a single line



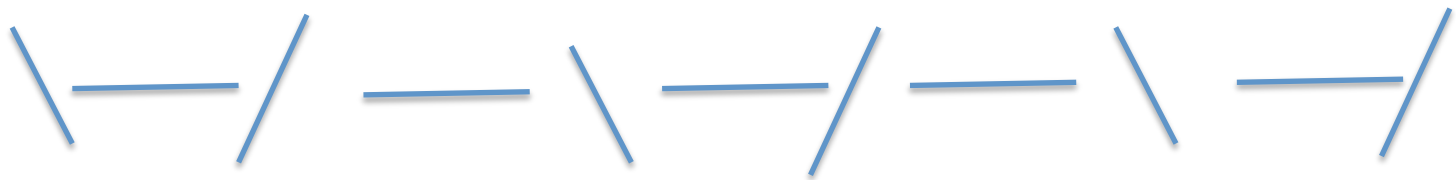
Tangent lines

Figure out where slope is zero, positive or negative



Tangent lines

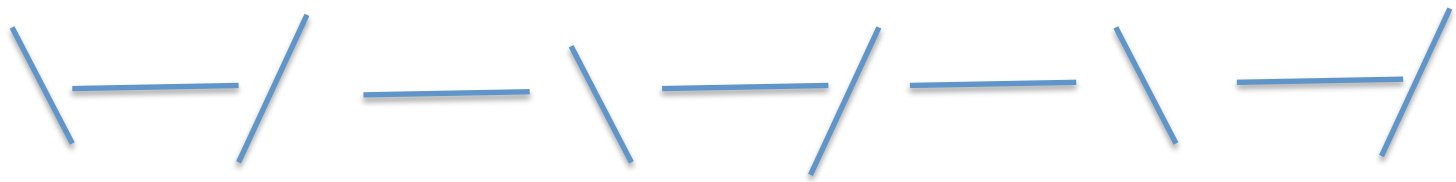
Figure out where slope is zero, positive or negative



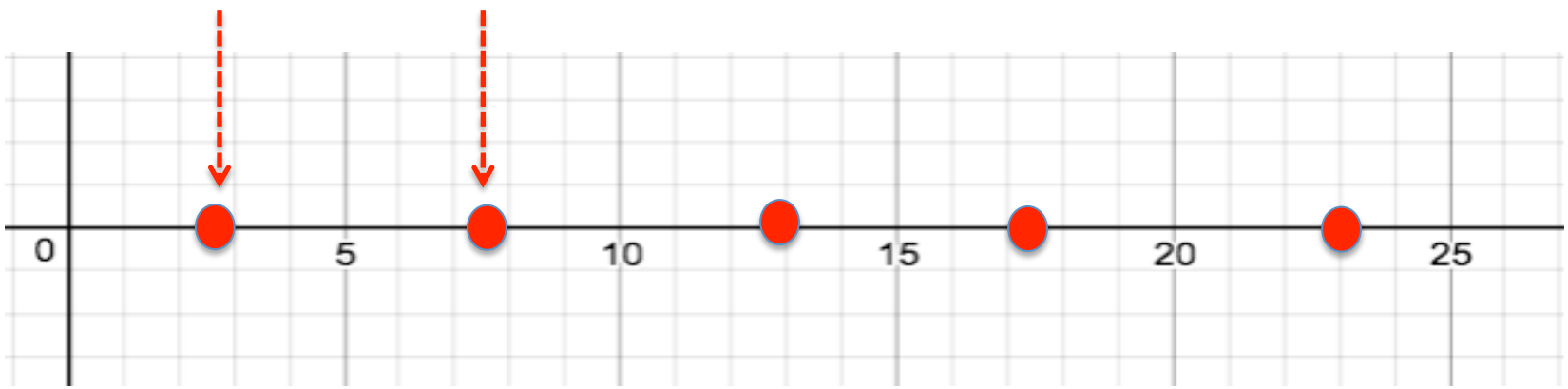
- 0 + 0 - 0 + 0 - 0 +

Tangent lines

Now sketch the slopes on a new graph, careful to align the places where it is zero

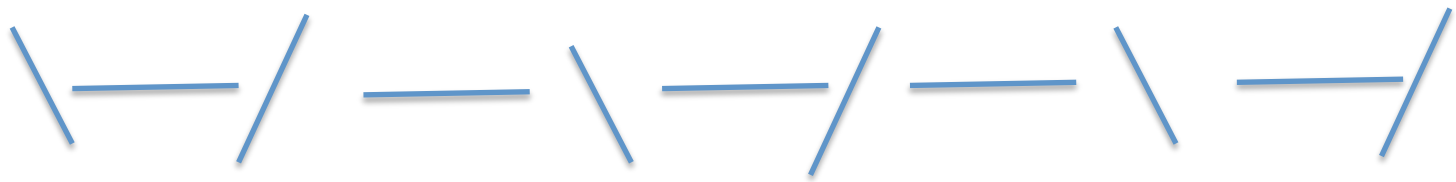


- 0 + 0 - 0 + 0 - 0 +

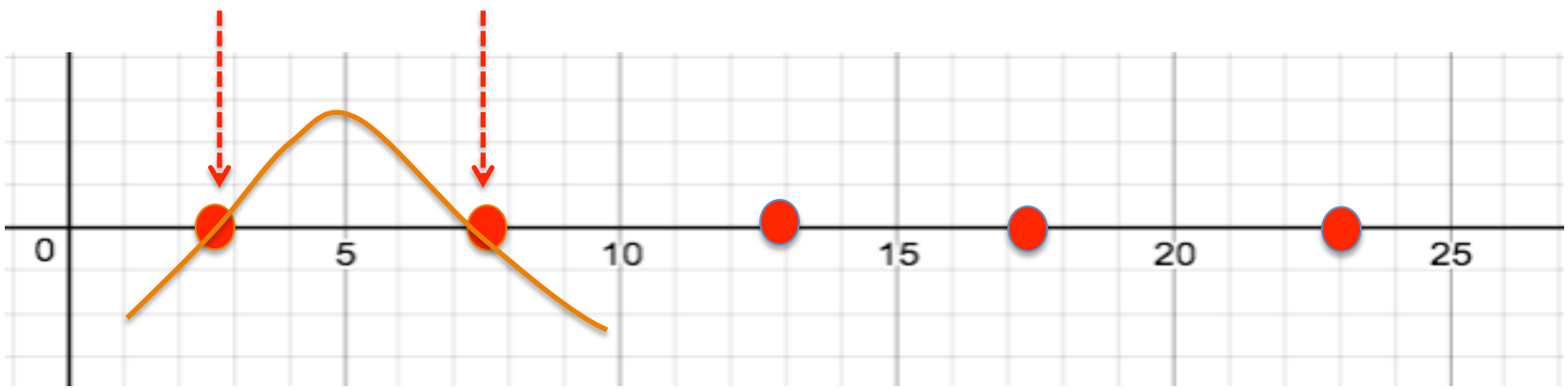


Tangent lines

Add the rest of the sketch

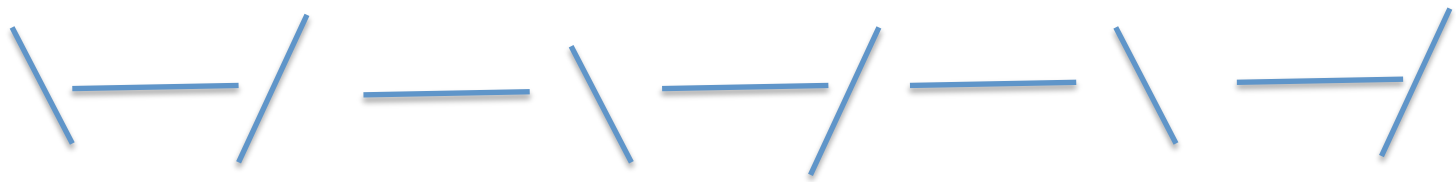


- 0 + 0 - 0 + 0 - 0 +

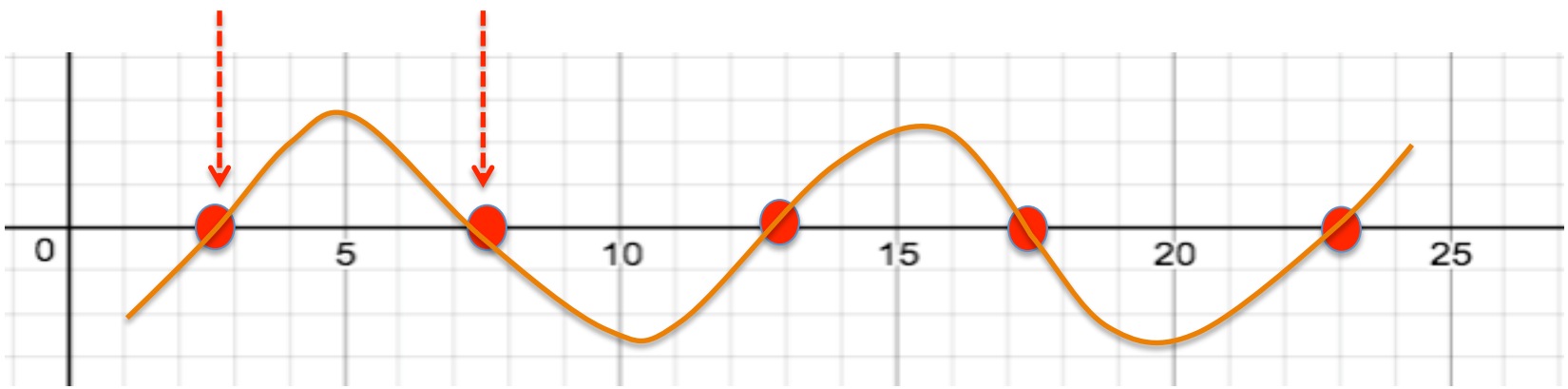


Tangent lines

Now we consider the slopes of the tangent lines



- 0 + 0 - 0 + 0 - 0 +



Rate of change ot the prey population

We can use desmos to graph the derviative

Derivative on desmos

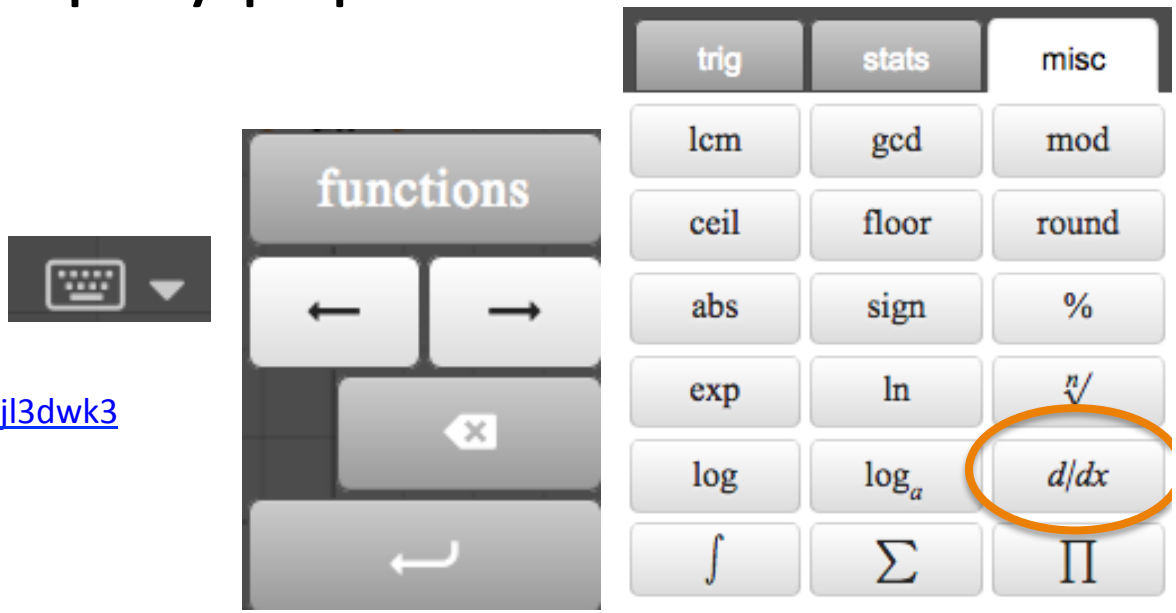
- Prey population

$$y_1 = 50 - 40\sin\left(2\pi\,\frac{t}{10}\right)$$

- Rate of change of prey population

3

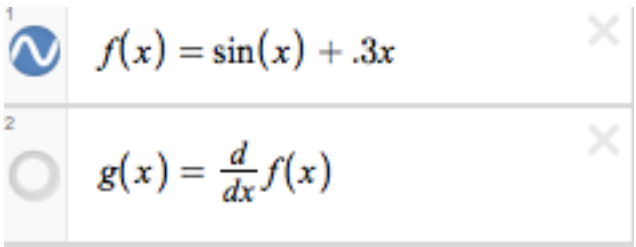
$$f = \frac{d}{dt}y_1$$

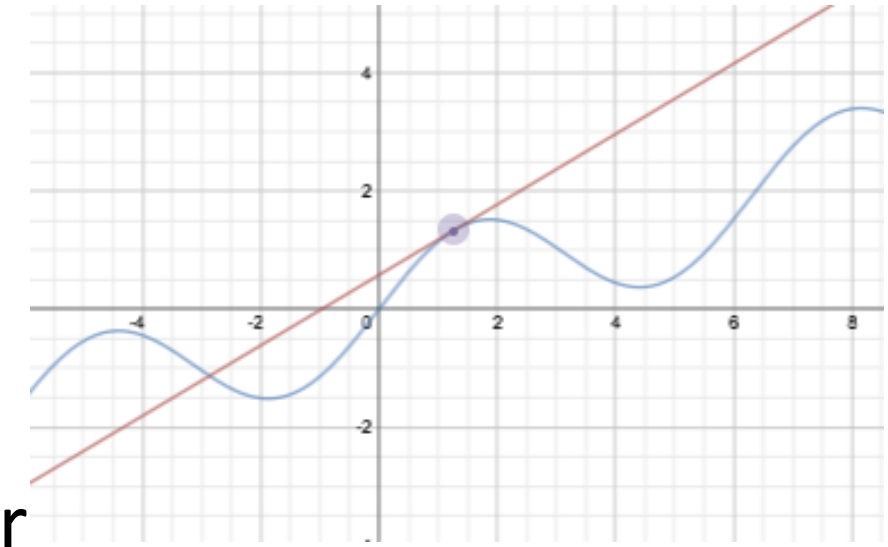


https://www.desmos.com/calculator/uenjl3dwk3

Desmos: tangent line

- https://www.desmos.com/calculator/fs3lx8gptb





- Experiment with slider
- Experiment with zoom!

Answers

1. D
2. D
3. A

Solutions

Practice calculation we did in class

Find the derivative of $f(x) = x^3$ using the defn

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \to 0} \frac{(\cancel{x^3} + 3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}\,2}) - \cancel{x^3}}{\cancel{h}}$$

$$= \lim_{h \to 0} 3x^2 + \underbrace{3xh}_{0} + \underbrace{h^2}_{0}$$

$$= 3x^2$$

Challenge:

Derivative of $y = x^n$ from the definition of the derivative:

See Appendix E.1

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^n - x^n}{h} \\
&= \lim_{h\to 0}\frac{h[c_1x^{n-1}+c_2x^{n-2}h+\cdots+c_nh^{n-1}]}{h} \\
&= \lim_{h\to 0}[c_1x^{n-1}+c_2x^{n-2}h+\cdots+c_nh^{n-1}] = c_1x^{n-1},
\end{aligned}
$$

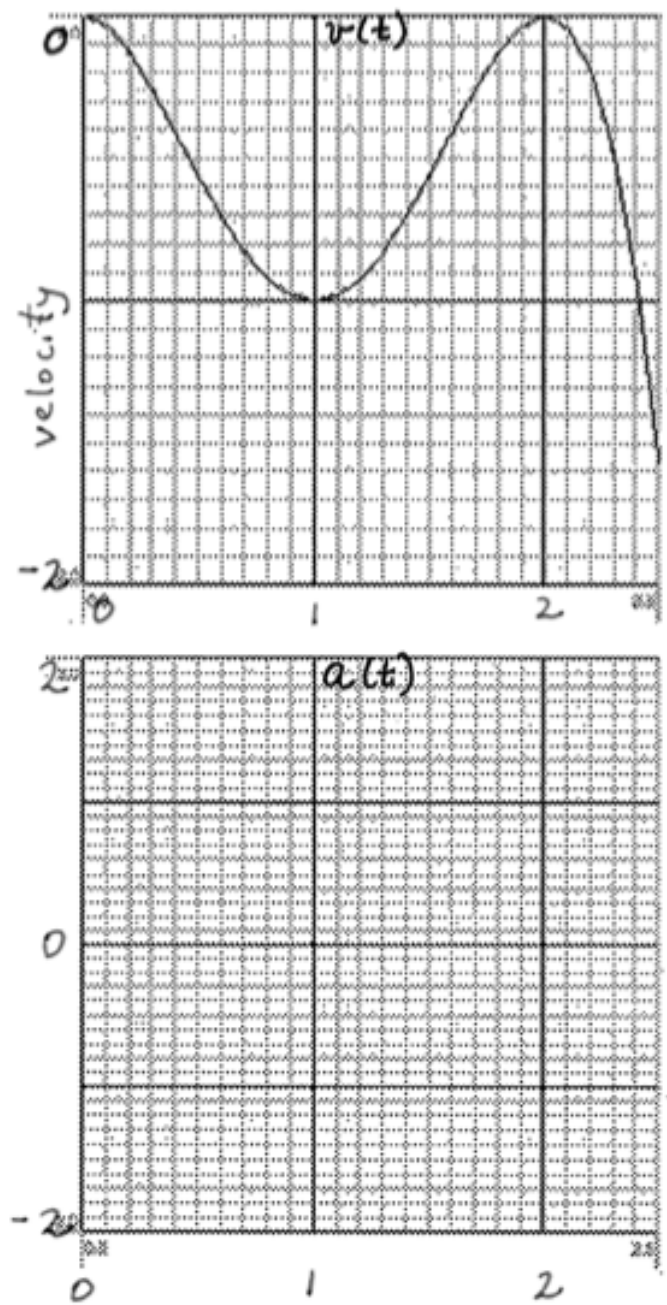
By the binomial theorem, $c_1 = n$, so $f'(x) = n\,x^{n-1}$

Problems to test your skill

A selection of problems from tests and exams to help you study concepts in this lecture

Related test problem

- Shown in the graph is the velocity of a particle. Use this to sketch the acceleration of the particle. (Hint: the acceleration is the derivative of the velocity)



Problem 2.20:

2.20. **Definition of the derivative.** Use the definition of derivative to calculate the derivative of the function

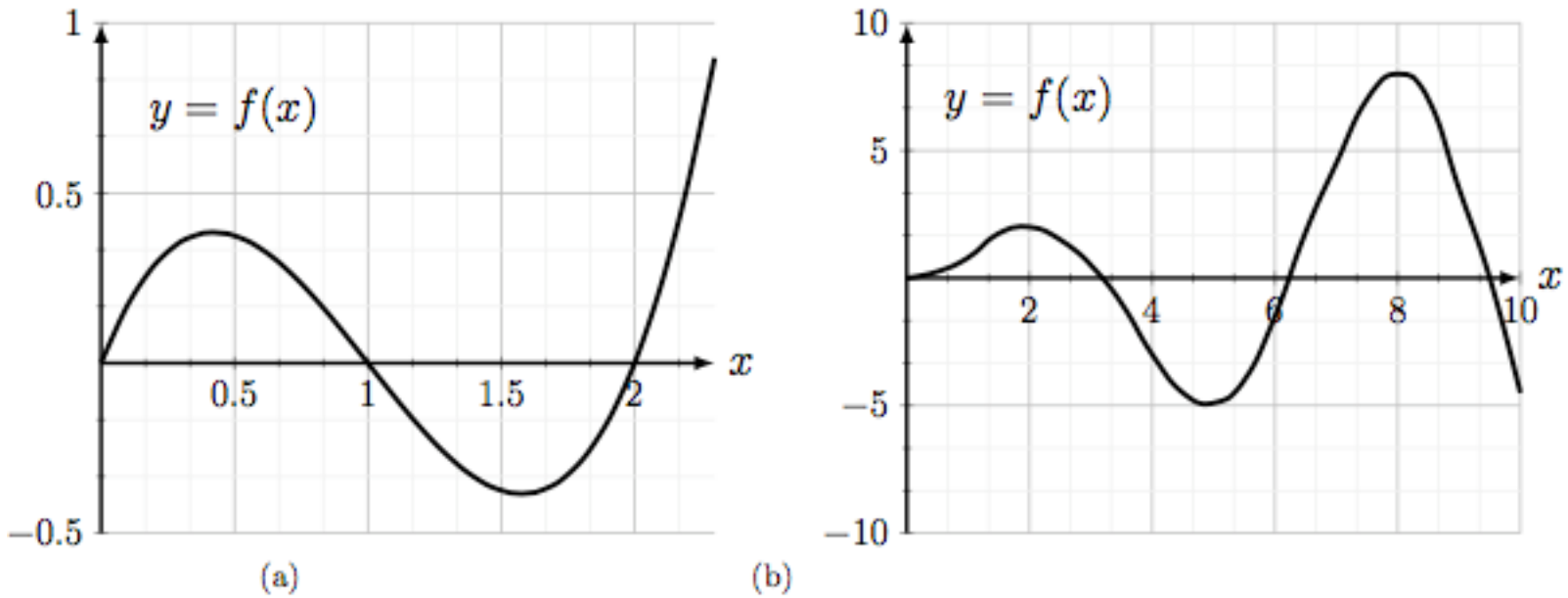
$$f(x) = \frac{1}{x+1}.$$

See P 67
prob 2.20

Exam-
type
question

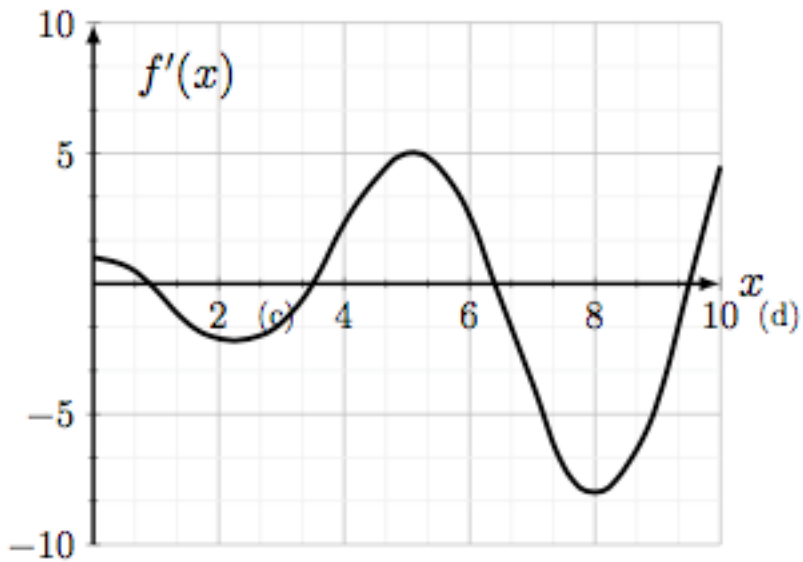
Problem 3.7:

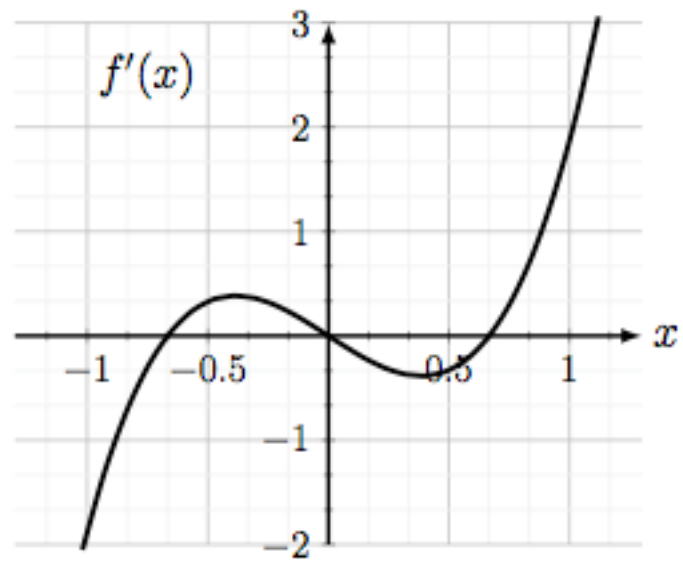
Graph the derivative of f(x)



See P 85
prob 3.7
a, b

Given the derivative f'(x), graph the original function f(x)





Problem 3.7:

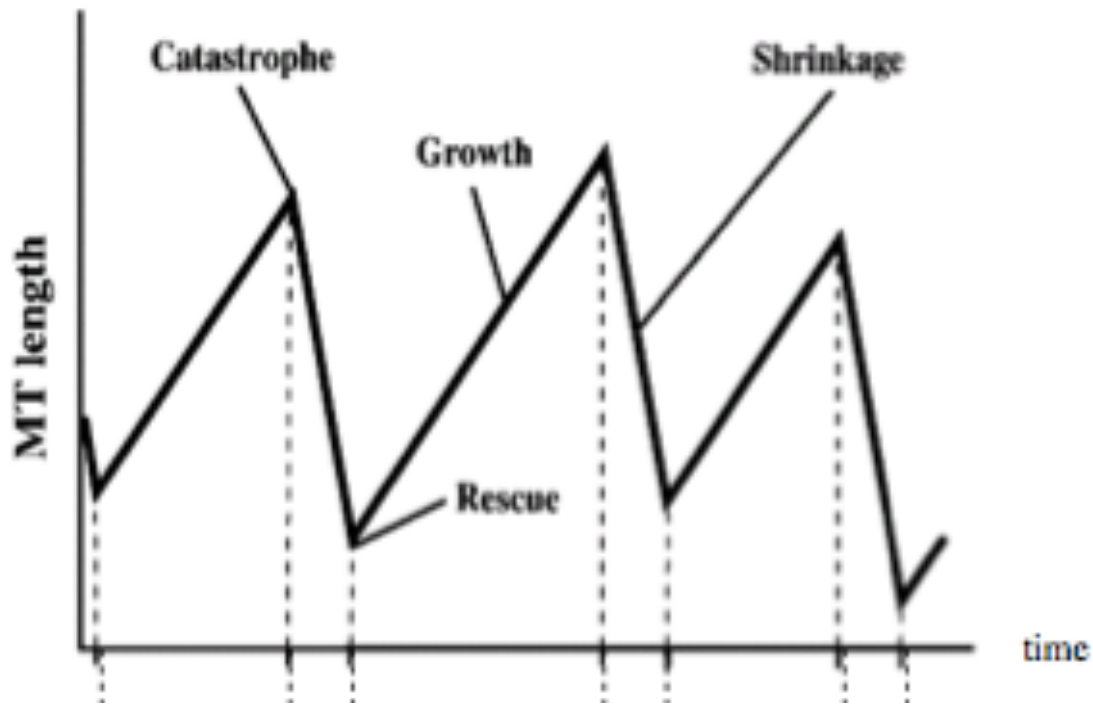
See P 85
prob 3.7
c, d

Related test problem (MT1, 2008)

Microtubules (MT) are biological polymers important in cell structure, cell division, and transport of material inside cells. The length of microtubules ("MT length") grows and shrinks as shown in the following figure from Janulevicius et al (2006) Biophys J 90: 788-798. Use this figure to draw a sketch of MT growth rate (i.e. rate of change of microtubule length per unit time) over the same time interval.

Which has a greater magnitude: the rate of shrinkage (per unit time) or the rate of growth (per unit time)?

Exam-type question



Solutions to previous problems

First try out the problems at the end of the last lecture slides. Only then should you “peek” at the answers.

2.13. **Secant and tangent lines.** Given the function $y = f(x) = 2x^3 + x^2 - 4$,

(a) find the slope of the secant line joining the points $(4, f(4))$ and $(4 + h, f(4+h))$ on its graph, where h is a small positive number, then

(b) find the slope of the tangent line to the curve at $(4, f(4))$.

Secant and tangent lines

- **Solution to Problem 2.13:**

$$f(4) = 2 \cdot 4^3 + 4^2 - 4 = 140.$$

The slope of secant line from $x = 4$ to $x = 4 + h$ is

$$\begin{aligned}
\frac{f(4+h) - f(4)}{(4+h) - 4} &= \frac{2(4+h)^3 + (4+h)^2 - 4 - 140}{h} \\
&= \frac{2(64 + 48h + 12h^2 + h^3) + (16 + 8h + h^2) - 4 - 140}{h} \\
&= \frac{2h^3 + 25h^2 + 104h}{h} \\
&= 2h^2 + 25h + 104 \text{ since } h \neq 0
\end{aligned}$$

The slope of the graph at $(4, 140)$ can be found by letting h go to zero in the last expression.

$$\lim_{h \to 0} (2h^2 + 25h + 104) = 104$$

A secant line is

(A) A line whose slope is instantaneous velocity

(B) A line connecting two points on a graph

(C) The same as an average velocity

(D) The same all along the curve

(E) Not sure.

Which of the below is an **average velocity**?

(A) Av veloc= $(v_1+v_2+v_3+v_4)/4$

(B) Av veloc = $(y_1+y_2+y_3+y_4)/4$

(C) Av veloc = $(y_1+y_2+y_3+y_4)/(t_1+t_2+t_3+t_4)$

(D) Av veloc = $(y_1+y_2)\ /(t_1+t_2)$

(E) Av veloc = $(y_2-y_1)/(t_2-t_1)$