

The Pacific Institute for the Mathematical Sciences,
International Graduate Training Center (IGTC)
Mathematical Biology Summer Course:
May 11-June 11, 2008

Organizer: Leah Edelstein-Keshet

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1 The IGTC Summer course

The International Graduate Training Center (IGTC) is a new program sponsored by the Pacific Institute for Mathematical Sciences (PIMS), a mathematical institute in Western Canada, to train graduate students in interdisciplinary research. The first IGTC graduate course in Mathematical Biology has been held at the University of British Columbia, Vancouver, Canada over a one month period (May 11-June 11, 2008). The course was open to current UBC students (as a normal three-credit course) as well as to students from elsewhere. It is equivalent to a three-credit graduate course. Students from Western Canada provinces get credit for this course based on the Western Dean's agreement.

2 Course number, hours, and other details

The IGTC summer course is an expanded version of Mathematics 561, "Topics in Mathematical Biology", a regular 3-credit UBC graduate course. The usual number of lecture hours per semester: 39. By comparison, the total number of lecture hours in IGTC summer school was 60. The total number of computer-lab hours (Matlab and XPP) was 18.

2.1 Prerequisites

Students were expected to have background in differential equations, linear algebra, and introductory modelling. No biological knowledge was assumed. The required biology was taught in parallel with the modeling and mathematical techniques.

2.2 Instructors

The course was be taught by the following instructors:

- Leah Edelstein-Keshet (Dept. of Mathematics, UBC): full time instructor; covered basic survey of Mathematical Biology with an emphasis on cellular biology.
- Prof Karl Hadeler (Dept. of Mathematics, Tübingen, and Visiting Professor, Arizona): Prof Hadeler was a 2-weeks visitor. He presented the mathematics of transport and diffusion equations and correlated random walks, and a variety of classical topics: species interactions, epidemiology, systems with quiescence, and delays.
- Prof Bard Ermentrout (Dept. of Mathematics, U Pittsburgh): Prof Ermentrout presented the theory of coupled oscillators and rhythmicity in the context of neurosciences. He presented several tutorials using the differential-equation and bifurcation analysis software, XPP.
- Prof Alex Mogilner (Dept of Mathematics, UC Davis) was a 1-week visitor. He lectured on the mathematical modeling associated with cell division, the role of microtubules, and the diverse deterministic and stochastic modeling approaches.
- Dr. Raibatak (Dodo) Das: a 2-day minicourse on fitting models to experimental data and parameter estimation

During the month-long course, there were three hours of lectures every morning and tutorials/computer labs in the afternoon several days per week. Student research seminars were held once per week, organized by and for the students. Most students in the course had an opportunity to present their research at such seminars. Every week, we also hosted an additional research-level guest speaker, with speakers from SFU and UBC such as Profs Michael Doebeli, Erikur Palsson, Yue Xian Li, Daniel Coombs, and Eric Cytrynbaum. Topics ranged from evolution and ecology to cell biology, immunology, and calcium dynamics.

Students were exposed to interesting biological questions at the forefront of current scientific research. They had the opportunity to learn how to use simulation software (such as XPP), methods of modeling and analysis, as well as fitting models to data. The course will provide practice at verbal and written communication in the form of a project. Credit will be based on problem sets and on a final project.

2.3 Keshet's Syllabus

Cell biology offers a rich area where concepts and methods of applied mathematics can come into play. The course started with simple enzyme-mediated chemical reaction kinetics, with specific dynamic properties (modeled by ODEs), focusing on recent applications to biochemical signalling in cells. Polymerization and actin cytoskeleton dynamics were discussed as specific examples. Dimensional analysis, and its usefulness as a tool for data analysis of microtubule polymerization were taught.

One week was spent on the formulation of stochastic models for molecular events, including the Gillespie simulation method, Langevin and Smoluchowski equations. The latter were applied to the derivation of the polymerization ratchet equations and the implications to growth of actin filaments in the cell. We then discussed several recent literature papers about the motion and shape of a simple cell, the keratocyte, illustrating the formulation of the model, its scaling, simplification and analysis.

We studied reaction-diffusion systems, illustrating both (Turing) pattern and wave-based phenomena, with applications to the problem of gradient sensing and directional selection in a polarizing cell.

2.4 Prof Mogilner's course description

In the last decade, cell biology has been undergoing revolutionary change, in no small part due to rapid accumulation of quantitative data and development of mathematical and computational models that helped to make sense of these data. As a result, mechanistic understanding of molecular processes in the cell increased dramatically. In the course of five lectures, Prof Mogilner used recent research papers to examine a few cell biological phenomena (i.e. biochemical signaling, cell cycle, cell division, intracellular transport) to understand what was the logic of the controlled experiments, how the data was used to formulate and test the models, and how the models helped to make sense of the data. Basic applied mathematics tools were used (ODEs, PDEs, elementary probability), and Matlab simulations suitable for studying such problems were presented.

2.5 Prof. Haderer's Course description:

Bacteria perform peculiar movements and control size and shape of their colonies by "quorum sensing". They "want" to optimize their environment, e.g. by "chemotaxis", and they "want" to spread. What are the mechanisms? Are movements and growth patterns arbitrary or governed by laws for the individual and the population? Can we cast these laws into equations with few parameters which can be identified in experiments? Similar questions can be posed for animal and plant species and for the spread of infectious diseases (West Nile virus is a recent example).

A suitable framework are reaction diffusion equations which exhibit biologically and mathematically challenging phenomena: patterns, localization (spikes), blow up, fronts and pulses. These occur also in refined models like damped wave equations, reaction transport equations, and Langevin equations. We demonstrated phenomena and establish connections between levels of complexity, e.g. by diffusion approximation. We asked whether a species is stable within an ecosystem (persistence) and how a species like the zebra mussel can invade (it took 150 years to get from England to North America).

We ask whether the dynamics of a system is determined by its components, when all components are similar (lattice dynamics) and when components represent quiescent or dormant phases. Examples are microbes (spores), cells ("most cells in the body are quiescent"), plants (seed banks) and mammals (hibernation).

Rather than modeling the size of fishes or rodent colonies by size classes one can use first order partial differential equations which allow dependence on individual growth rates and total population size, but mostly not stochastic variation. We incorporate stochastic behavior by diffusion (mathematically: viscosity approach to conservation laws).

Modeling infectious diseases is a most important application: vaccination, education and quarantine policies, effect of social structure, core groups and small reservoirs, demographic impact, and spread in space, as well as parameter identification from field data.

2.6 Prof. Ermentrout's course description

In this set of lectures Prof Ermentrout discussed the modeling and analysis of rhythmic phenomena in biology. He started with models of oscillations including ODE models, PDE models and delay equations, laying out the necessary conditions that are required for oscillations. He discussed the underlying geometry of mechanisms from which a system goes from a stable fixed point to a periodic orbit. He then switched to questions concerning the interaction of intrinsic oscillators, that is, what kinds of behavior can we expect when oscillators are coupled. The main approach was the reduction to "phase models" in which each oscillator is reduced to a one-dimensional variable on a circle. The interactions between the oscillators as well as the topology of the connections determines the stable patterns which are possible. Prof Ermentrout showed how both the intrinsic properties as well as the time scales of interactions combine to determine the stability of patterns in networks. In addition to synchronization behavior, he provided conditions for various types of wave-like behavior including target waves, spiral waves, and complex dynamics. Applications to neural and cellular phenomena as well as many computer exercises in XPPAUT were included.

3 Course Credit and Grading

The course is assigned 3 full credits, typical of a full-semester course at the University of British Columbia. Most students are taking the course as PASS/FAIL.

Students receiving a certificate of completion are considered to have taken the course with a grade of PASS.

For students requiring a numerical grade, the following grading scheme has been used to assign a final grade:

Evidence of work on and comprehension of exercises and homework assignments: 40%. Participation and completion of computer labs: 10%. Participation in discussions and evidence of learning: 10%. Presentation and quality of a research seminar: 20%. Presentation and quality of final project: 20%. In certain cases, up to 5% extra credit was awarded to student volunteers who actively participated in organizational aspects of the course.

4 Other details

Details have been posted at:

<http://www.pims.math.ca/science/2008/08ssigtcmb/>