

## SCIENCE ONE, MATHEMATICS - HOMEWORK #4

*Due 10AM, Friday, Nov. 21*

PROBLEM 1. The radial probability density function for the ground state of the hydrogen atom is

$$P(r) = \frac{4r^2}{a^2} e^{-2r/a}, r \geq 0,$$

where  $a > 0$  is a constant. Draw a graph of the function identifying its relative maximum and points of inflection.

PROBLEM 2. Identify all extrema and inflection points of  $f(x) = x + \sin x$ , then sketch its graph. Make sure your sketch shows all significant features of the function.

PROBLEM 3. We want to approximate  $\ln x$  for  $x \geq 1$  using a Taylor polynomial  $T_n(x)$  centered at  $a = 1$ . The approximation must be accurate to two decimal places (this means that the remainder satisfies  $|R_n(x)| \leq 0.005$ ).

- (a) If we use the degree 2 Taylor polynomial  $T_2(x)$ , how close does  $x$  have to be to 1 to guarantee two decimal place accuracy?
- (b) Which smallest  $n$  guarantees two decimal place accuracy for all  $x$  in  $[1, 2]$ ?

PROBLEM 4. Find the limit

$$\lim_{x \rightarrow 0} \frac{1}{\ln(1+x)} - \frac{1}{\tan x}.$$

(Hint: replace  $\ln(1+x)$  and  $\tan x$  by the first few terms of their Taylor series.)