SCIENCE ONE, MATHEMATICS - HOMEWORK #2

Due 10AM, Friday, Oct. 17

PROBLEM 1. If f is a differentiable function and $g(x) = x^2 f(x)$, use the definition of derivative as a limit to show that $g'(x) = x^2 f'(x) + 2xf(x)$.

PROBLEM 2. Show that if $b > 1/2$, there are three straight lines through $(0, b)$, each of which is normal to the curve $y = x^2$. How many such lines are there if $b = 1/2$? If $b < 1/2$? Explain.

PROBLEM 3. Let $m > 0$ and $k > 0$ be given constants (m is the mass of a particle attached to a spring that has a Hooke's Law constant K), and consider the problem of finding the displacement $x(t)$ that satisfies the differential equation

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x, \quad 0 < t < \infty,$$

and satisfies at $t = 0$ the two initial conditions

$$x(0) = x_0, \quad x'(0) = v_0.$$

The solution can be expressed as

$$x(t) = A\cos(\omega t + \phi_0), 0 \le t < \infty.$$

Suppose $m = 3$ kg and $k = 40$ N·m^{-1}. Find A, ω and ϕ_0 (including their units) if the initial position and velocity of the mass is

(a)$x_0 = 3$ m, and $v_0 = 1$ m· s^{-1}.
(b)$x_0 = 3$ m, and $v_0 = -1$ m· s^{-1}.
(c)$x_0 = -3$ m, and $v_0 = 1$ m· s^{-1}.
(d)$x_0 = -3$ m, and $v_0 = -1$ m· s^{-1}.

PROBLEM 4. Find all strictly positive values of the constant λ such that the boundary value problem

$$\frac{d^2u}{dx^2} = -\lambda u, \quad 0 < x < \pi,$$
$$u(0) = 0, \quad u'(\pi) = 0$$

has nontrivial solutions, and also give the corresponding solutions $u(x)$.