Let \( k = \mathbb{C} \) in all problems.

**Problem 1.** (1) Prove that the four curves in \( \mathbb{A}^2 \):
\[
V(x^2 \pm y^2 \pm 1)
\]
all have affine coordinate rings isomorphic to \( \mathbb{C}[z, z^{-1}] \), hence the curves are isomorphic.

(2) Describe an isomorphism \( f : V(x^2 - y^2 - 1) \to V(x^2 + y^2 + 1) \).

(3) Show that the curves \( V(x - y^2 - 1) \) and \( V(x^2 - 2xy + y^2 - 1) \) are not isomorphic to the curves above or to each other. Find the affine coordinate rings of these two curves.

**Problem 2.** Let \( X = \mathbb{A}^1 \{a, b, c\} \), where \( a, b, c \) are distinct.

(1) Describe all morphisms from \( X \) to \( \mathbb{A}^1 \). Which ones are injective?

(2) Find \( a, b, c \) such that \( X \) is not isomorphic to \( \mathbb{A}^1 \{0, 1, 2\} \).

**Problem 3.** Let \( X \subseteq \mathbb{A}^2 \) be the complement of 4 distinct points, no three of which lie on a line. Is it true that any two such \( X \) are isomorphic? Equivalently, is any such \( X \) isomorphic to the complement of \( (0,0), (0,1), (1,0), (1,1) \)? (There are many more isomorphisms \( f : \mathbb{A}^2 \to \mathbb{A}^2 \). In addition to linear maps and translations, there are also maps of the form \((x, y) \mapsto (x, y + f(x))\) where \( f(x) \) is nonlinear.)

**Problem 4.** Consider \( X = \mathbb{C}^x = \mathbb{C} \sim \{0\} \) with the usual metric topology. Let \( L \) be the sheaf of functions on \( X \) such that \( L(U) \) is the set of continuous \( \mathbb{C} \)-valued functions \( f(z) \) on \( U \) satisfying \( \exp(f(z)) = z \). You may assume that this is a sheaf.

(1) Show that this sheaf is locally constant. Every point \( x \in X \) has a neighborhood \( U \) such that \( L(U) \simeq \mathbb{Z} \) as a set.

(2) Show that \( L \) has no global sections, \( L(X) = \emptyset \).

**Problem 5.** Let \( X \subseteq \mathbb{A}^n \) be a closed subset. Define the sheaf \( I_X \) on \( \mathbb{A}^n \) with sections \( I_X(U) \) those functions on \( U \) that vanish on \( X \cap U \).

(1) Prove that this is a sheaf. (Explain why the sheaf axiom holds.) This is called the ideal sheaf of \( X \).

(2) Let \( X = \{(0,0)\} \subseteq \mathbb{A}^2 \). Find the stalks \( I_{X,x} \) for all points \( x \in \mathbb{A}^2 \). You may describe these stalks in terms of stalks \( O_{\mathbb{A}^2,x} \).