1. (1) The number of \( \xi \in \text{temp}(K\overline{F}) \) is \( \text{deg}(K/F) \).
   Each \( \xi \) gives an embedding \( K \rightarrow \overline{F} \). To lift \( \xi \) to an embedding \( \gamma : E \rightarrow \overline{F} \) there are \( \text{deg}(E/K) \) choices.
   Thus, there are \( \text{deg}(K/p) \cdot \text{deg}(E/K) \cdot \text{deg}(E/F) \) choices for \( \gamma \).
   This implies that \( E/F \) is separable.

(2) Let \( \alpha \in E \) and \( f(x) \) the minimal polynomial of \( \alpha \) over \( F \).
   Then \( f(x) \) has distinct roots. If \( \alpha \in K \), then \( f(x) \) itself
   has distinct roots, hence \( K/F \) is separable. The polynomial \( f(x) \)
   may split into irreducible factors in \( K[x] \). One of these
   factors is the minimal polynomial of \( \alpha \) over \( K \). The
   factor still has distinct roots.

2. (1) Example
   \[
   \begin{array}{c|c|c}
   \mathbb{Q}(\sqrt[3]{5}) & \text{normal} & \text{not normal, the conjugate} \\
   \mathbb{Q}(\sqrt[3]{5}) & \text{normal} & i \sqrt[3]{5}, i, \frac{1}{2} \sqrt[3]{4}, \text{is not in the extension.} \\
   \mathbb{Q} & \text{normal} & \\
   \end{array}
   \]

(2) If \( \alpha \in E \), \( f(x) \) is its minimal polynomial, then all roots of
   \( f(x) \) lie in \( E \). The minimal polynomial of \( \alpha \) over \( K \) is a
   factor of \( f(x) \), hence all its roots lie in \( E \).

(3) \( \mathbb{Q}(\sqrt[3]{2}, \xi) \) \( \xi \): 3rd root of 1.

\[
\begin{array}{c}
\mathbb{Q}(\sqrt[3]{2}) \\
\mathbb{Q}(\sqrt[3]{2}) \\
\mathbb{Q} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\mathbb{Q}(\sqrt[3]{2}) & \text{normal, splitting field of } x^2 - 2, & \\
\mathbb{Q} & \text{not normal, } \sqrt[3]{2 \sqrt[3]{2}} \text{ is conjugate to } \sqrt[3]{2}, & \text{not in the field.}
\end{array}
\]
3. (1) Let \( \alpha \) be a root of \( f(x) = x^p - \alpha \).

and the minimal polynomial of \( \alpha \) over \( F \) is a factor of \( f(x) \).

However, in \( \overline{F}[x] \), \( f(x) \) splits

\[ f(x) = (x - \alpha)^p, \]

so \( \alpha \) is the only root of its minimal polynomial.

(2) Every \( \alpha \in E \) has only one conjugate, \( \alpha \) itself. \( \sigma \in \text{Gal}(E/F) \)

must map \( \alpha \) to one of its conjugates, hence \( \sigma = id_E \).

(3) Let \( K \) be the field generated by \( S \), \( F \subseteq S \subseteq K \subseteq E \).

But all elements of \( K \) are separable, hence \( K \subseteq S \).

Clearly \( F \subseteq S \) is separable because every \( \alpha \in S \) is separable over \( F \).

Let \( \alpha \in E \). We show that \( \alpha^{p^m} \in S \) for some \( m \).

Let \( f(x) \in F[x] \) be the minimal polynomial of \( \alpha \).

If \( f'(x) = 0 \), we can write \( f(x) = h(x^{p^k}) \), and repeat with \( h \).

In the end we get

\[ f(x) = g(x^{p^k}), \quad g'(x) \neq 0, \quad g(x) \text{ is separable}. \]

Since \( \alpha \) is a root of \( f(x) \), \( \alpha^{p^m} \) is a root of \( g(x) \)

and hence separable.