1. Calculate the Fourier coefficients \((c_n, a_n, b_n)\) for the triangle function

\[
f(t) = \begin{cases} 
2t & \text{if } 0 \leq t \leq 1/2 \\
2 - 2t & \text{if } 1/2 \leq t \leq 1 
\end{cases}
\]

and show that the Fourier series decomposition of \(f(t)\) may be written

\[
f(t) = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{4}{\pi^2 n^2} \cos(2\pi nt) = \frac{1}{2} - \sum_{n=0}^{\infty} \frac{4}{\pi^2 (2n+1)^2} \cos(2\pi(2n+1)t)
\]

What does Parseval’s formula say in this case?

2. Modify the file \(ftdemo1.m\) so that it plots the partial sums of the Fourier series in the previous question. Hand in the code and a plot of the partial sums with 1, 2, 5 and 10 non-zero terms.

3. Calculate the Fourier coefficients \((c_n, a_n, b_n)\) for the half sine wave

\[
f(t) = \sin(\pi t) \quad \text{for } 0 \leq t \leq 1
\]

and show that the Fourier series for \(f(t)\) can be written

\[
f(t) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{1 - 4n^2} \cos(2\pi nt)
\]

4. Calculate the Fourier coefficients \((c_n, a_n, b_n)\) for the function

\[
f(t) = t^2 - 1 \quad \text{for } -1 \leq t \leq 1
\]

and show that the Fourier series for \(f(t)\) can be written

\[
f(t) = -\frac{2}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos(n\pi t)
\]

5. Show that the Fourier series of \(f(t) = e^t\) on the interval \(-\pi \leq t \leq \pi\) is

\[
f(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{1}{1 - in}(e^{(1-in)\pi} - e^{-(1-in)\pi})e^{int}
\]

Deduce that

\[
\sum_{n=1}^{\infty} \frac{1}{1 + n^2} = \frac{1}{2}(\pi \coth \pi - 1).
\]