MATH 300 - SECTION 201
EXAM #1

Name:
Student ID:

Exam rules:
• No calculators, open books or notes are allowed.
• There are 7 problems in this exam.
Problem 1. (a) Write the following number in the form $a + bi$:
\[
\frac{i}{2} + \frac{2}{i}.
\]

(b) Write the following number in the polar form $r e^{i\theta}$:
\[
\frac{1 + i}{ie^{2+\pi i}}.
\]
PROBLEM 2. (a) Sketch the set of all complex numbers $z$ such that
\[ \text{Arg}(z) \geq \frac{\pi}{2} \text{ and } 1 \leq |z| \leq 2. \]

(b) Sketch the image of the set in part (a) by the function $f(z) = z^3$. 
Problem 3. (a) Find all fourth roots of $z = 9$. Leave your final answers in the form $a + bi$.

(b) Find all solutions to the equation

$$z^2 - 2z + 1 - 2i = 0.$$  

Your final answers should be in the form $a + bi$. 
Problem 4. Let
\[ v(x, y) = x^3 + 3xy(1 - y). \]
(a) Prove that \( v(x, y) \) is harmonic everywhere.

(b) Find a function \( u(x, y) \), such that \( f(x + iy) = u(x, y) + iv(x, y) \) is analytic.
PROBLEM 5. This problem can be solved using Cauchy-Riemann equations. You do not need to calculate limits here.
(a) Find where the function
\[ f(z) = z \cdot \overline{z} \]
is differentiable.

(b) Show that the function
\[ f(z) = \text{Im}(z) + i\text{Re}(z) \]
is nowhere differentiable.
**Problem 6.** Let \( f(x + iy) = u(x, y) + iv(x, y) \) be an analytic function.

(a) Find real valued functions \( s(x, y) \) and \( t(x, y) \), such that

\[
\frac{1}{f'(x + iy)} = s(x, y) + it(x, y).
\]

Express \( s \) and \( t \) in terms of partial derivatives \( u_x, u_y, v_x, v_y \). (Here \( u_x = \partial u/\partial x \) and so on.)

(b) Express the functions \( s \) and \( t \) in part (a) in terms of partial derivatives \( u_x \) and \( u_y \) only.
Problem 7. Mark each statement TRUE or FALSE. Give a short explanation. (A statement is true if it is true for all $z, w$. It is false if it fails for some $z, w$.)

(a) $\text{Re}(z + w) = \text{Re}(z) + \text{Re}(w)$.

(b) $|z + w| \geq |z|$.

(c) $|\text{e}^z| = e^{|z|}$.

(d) The function

$$f(z) = z^{10} - iz + z\overline{z}^5$$

is nowhere analytic.