Math 300: Assignment #4:
Due: Friday, Oct. 16. in class.

1. Section 2.4: 10. (Hint: Use Cauchy-Riemann equations.)

2. Section 2.4: 12.

3. Section 2.5: 6, 8(b).

4. Section 2.5: 18. (This can be checked directly, using Cauchy-Riemann equations. If \( \phi \) is the real part of an analytic function \( f(z) \), then what is this new function in terms of \( f(z) \)?)

5. Let
\[
f(z) = 1 + z + z^2 + \cdots + z^7.\]
a. Write \( f(z) \) as a product of linear factors. (Hint: \((z - 1)f(z)\) has a simple form.)
b. Find all poles and their orders in
\[
\frac{1 + z^4}{f(z)}.
\]

6. Find the partial fraction decomposition of
\[
\frac{5z^4 + 3z^2 + 1}{(z + i)(z^2 + 2)}.
\]
(Hint: First apply division with remainder to the polynomials. To check your computations, all coefficients should be integers.)

7. Section 3.2: 7. (Use the definition of \( \sin z \) and \( \cos z \) in terms of exponentials.)

8. Section 3.2: 12.

9. Section 3.2: 14. (Check that \( f(z + P) = f(z) \), where \( f \) is the given function and \( P \) the period. To be precise, one should also prove that there is no smaller \( P \) that works, but you may skip this step.)

10. Section 3.2: 25. You can do parts a,b,c,d. Or you can explain why for any complex number \( C \) there exists a number \( z, |z| < 0.001 \), such that \( e^{1/z} = C \). (Hint: replace the problem of finding such \( z \) with the problem of finding \( 1/z \).)