

**SPECTRA OF SOL-MANIFOLDS  
SUMMER PROJECT, 2010**

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An  $n$ -dimensional manifold is a geometric object that locally looks like the Euclidian space  $\mathbb{R}^n$ , but can have a complicated global structure. For instance, a simple curve is a 1-dimensional manifold; a sphere, a torus (shaped like a donut) and other surfaces you have encountered in multivariable calculus are examples of 2-dimensional manifolds. The Laplacian is a partial differential operator that is ubiquitous in almost any branch of the basic and engineering sciences. Eigenvalues of the Laplacian on a manifold contain important geometric information about the manifold. For example the notes produced by a drum are the eigenvalues of the Laplacian on the drum. A famous question of Kac asks “Can you hear the shape of a drum?” which means whether one can correctly guess the geometry of the drum based solely on the notes that it produces. *This project is about the distribution of eigenvalues or characteristic frequencies of the Laplacian on a manifold, an exciting area that lies at the interface of two rich and deep fields: Riemannian geometry and partial differential equations.*

The purpose of this project is to conduct a numerical study of the eigenvalue counting function for a special class of 3-dimensional manifolds (namely Sol-manifolds), and in the process get an introduction to Riemannian geometry and partial differential equations on manifolds. The project consists of two distinct components: a reading course on Riemannian geometry and spectral theory on manifolds, running concurrently with a numerical implementation of the eigenvalue counting function. A working knowledge of MATLAB, Mathematica and computer programming (in, for example, Fortran or C/C++) is required. Familiarity with real analysis (MATH320/321) and ordinary differential equations is a strong plus. This will be a group project with two or three students, so applicants must be committed to working in a team.