Math 120 Homework 9

- Due Monday Nov 28 at start of class.
- If your homework is longer than one page, **staple** the pages together, and put your name on each sheet of paper. **Homework that is not stapled will loose 1 point.**
- Each homework problem should be correct as stated. Occasionally, however, I might screw something up and give you an impossible homework problem. If you believe a problem is incorrect, please email me. If you are right, the first person to point out an error will get +1 on that homework, and I will post an updated version.

L'Hopital's rule

1. (5 points) Consider $\lim_{x\to\infty} \frac{\cosh x}{\sinh x}$. Prove that this is of the indeterminant form ∞/∞. Can L'Hopital's rule be used to compute the limit? If so, use L'Hopital's rule to compute the limit. If not, prove it.

2. Use L'Hopital's rule to prove the following

(a) (4 points) Prove that if f and g are polynomials with $f(x) = a_n x^n + \ldots + a_0$ and $g(x) = b_n x^n + \ldots + b_0$, with $a_n \neq 0,\ b_n \neq 0$, then

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = a_n/b_n.$$

(b) (3 points) Prove that if f and g are polynomials with $\deg(g) > \deg(f)$, then

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0.$$

3. (5 points) Give an example of two functions $f(x)$ and $g(x)$ with the following properties:

- $\lim_{x\to\infty} f(x) = 1$
- $\lim_{x\to\infty} g(x) = \infty$
- $\lim_{x\to\infty} f(x)^{g(x)} = 5$

Prove that your example is correct.

Anti-Derivatives

4. (4 points) Prove that if F_1 and F_2 are anti-derivatives for the function f on the interval (a, b), then $F_1(x) - F_2(x)$ is a constant.

5. a (3 points). Prove that the function

$$f(x) = \begin{cases} 1, & x = 0, \\ 0, & x \neq 0. \end{cases}$$

does not have an anti-derivative on the interval $(-1, 1)$.

(b, 4 points). Prove that the function

$$g(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geq 0. \end{cases}$$

Has an anti-derivative on any interval (a, b) that does not contain 0, and does not have an anti-derivative on any interval (a, b) that contains 0.

First order differential equations

6 (5 points) Solve the initial value problem

$$\begin{aligned} &y'(x) + xy(x) = 0, \\ &y(0) = 1. \end{aligned}$$

7. (2 points) Solve the initial value problem

$$\begin{aligned} &y'(x) = \sin(x), \\ &y(0) = 17. \end{aligned}$$