Math 120 Homework 5

- Due Friday October 21 at start of class.
- If your homework is longer than one page, **staple** the pages together, and put your name on each sheet of paper. **Homework that is not stapled will loose 1 point.**
- Each homework problem should be correct as stated. Occasionally, however, I might screw something up and give you an impossible homework problem. If you believe a problem is incorrect, please email me. If you are right, the first person to point out an error will get +1 on that homework, and I will post an updated version.

Newton iteration

1. (5 points) Use Newton's method with an initial guess $x_1 = 1$ to approximate the (positive) fourth root of 2, and show the computations. It's enough to iterate 4 times, and feel free to use a calculator.

2. Let $f(x) = x^3 - 2x + 2$.

a. (2 points) Prove that there is at least one point $x \in \mathbb{R}$ for which $f(x) = 0$ (i.e. f has at least one root)

b. (2 points) Let $x_1 = 0$. Use Newton's method to compute the first few iterates $x_2,\ x_3,\ x_4$.

c. (5 points) Prove that for each $n = 1, 2, 3, \ldots$, $|f(x_n)| \geq 1$, where x_0, x_1, etc are the points from part b. (you could do a proof by induction, but you don't have to).

Remark: problem 2c shows that if you choose a bad starting guess x_1, then Newton's method might not find a root of f.

Intermediate Value Theorem & Rolle's theorem

3. (5 points) Let f be a function that is continuous on the interval $[0, 1]$, and suppose $0 \leq f(x) \leq 1$ for all $x \in [0, 1]$. Prove that there is at least one point $c \in [0, 1]$ with $f(c) = c$.

4. (5 points) Use Rolle's theorem to prove that for each value of b, if we define $f(x) = x^3 - 3x + b$ then there is at most one point $x \in [-1, 1]$ with $f(x) = 0$.

Mean Value theorem

(This will be covered in lecture on Monday)

5. (5 points) Let f be a function that is continuous on the interval $[a, b]$ and differentiable on the interval (a, b). Suppose that there exists $M > 0$ so that for each $x \in (a, b)$, we have $|f'(x)| \leq M$. Prove that $|f(b) - f(a)| \leq M|b - a|$.