# Department of Mathematics
# University of British Columbia
# MATH 300
# FINAL EXAM -Term 2, Winter 2007-2008

**Time: 7:00pm- 9:30pm** **Date:** Apr. 24, 2008.

Name: ____________________ Section Number: ______

I.D. Number: ______________ Signature: ____________________

THERE ARE 13 NUMBERED PAGES ON THIS EXAM. YOU MAY USE THE LAST FOUR PAGES EITHER TO WRITE YOUR ANSWERS OR FOR ROUGH WORK. YOU MAY TEAR THEM OFF TO USE. YOU ARE NOT ALLOWED TO USE CALCULATORS, NOTES OR BOOKS TO AID YOU DURING THE TEST.

| Question | **Mark** | **Out of** |
|---|---|---|
| 1 | | 10 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 20 |
| 6 | | 10 |
| 7 | | 10 |
| 8 | | 10 |
| Total | | 90 |

[10] 1. a) Determine where the function $f(x+iy) = e^x + ie^{2y}$ is differentiable *and* where it is analytic.

b) Let $f(z)$ be entire. Prove that $\overline{f(\bar{z})}$ is also entire.

[10] 2. Determine the domain of analyticity of:

a) $f(z) = \frac{1}{e^z+2}$

b) $f(z)$=Log(Log$(z) - i\frac{\pi}{2}$)

[10] 3. a) Find the first three coefficients, $a_0, a_1, a_2$, in the Taylor series of $\frac{1}{1+e^z}$ around $z_0 = 0$. What is the radius of convergence? (give a reason for your answer)

b) Let $\sum_{n=0}^{\infty} a_n z^n$ be the Taylor series of some analytic $f(z)$ at 0. Show that if $f$ is even (i.e., $f(z) = f(-z)$ for all $z$), then $a_n = 0$ for all odd $n$.

[10] 4. Find two different Laurent series for the function $f(z) = Log(z) + 1/(z - i)$ in annuli centered at $z_0 = 2i$ and specify the domain of convergence for each series.

[20] 5. Calculate each of the following contour integrals (assume every closed contour is oriented counter-clockwise)

a) $\int_C \bar{z}^2 dz$ where $C$ the line segment from 0 to $1+2i$.

b) $\int_C \frac{Log(z)}{z^2+9} dz$ where $C$ is the circle $|z-4i|=3$.

c) $\int_C \sin(3/z) dz$ where $C$ is the unit circle $|z|=1$.

d) $\int_C \frac{1}{(z-\alpha)(z-\beta)} dz$ where $C$ is a counter clockwise circle and $\alpha$ and $\beta$ lie strictly inside $C$.

[10] 6. Find

$$\int_0^{\infty} \frac{x^2+1}{(x^2+4)^2}dx$$

and justify your answer.

[10] 7. In each case below, find an example of a function $f(z)$ satisfying:

a) The Taylor series at $z = 3$ has radius of convergence $= 5$, and $f(2) = 2$.

b) $f(z)$ has an essential singularity at $z = 0$, a pole of order 1 at $z = 1$, a pole of order 2 at $z = 2$ and is analytic everywhere else.

c) $\int_C f(z)dz$ is equal to $i$ when $C$ is the circle $|z| = 1$, and it is equal to 1 when $C$ is the circle $|z| = 3$.

[10] 8. a) Let $f(z)$ be an entire function satisfying

$$|f(z)| \geq 1$$

for all $z$. Show that $f(z)$ must be a constant function.

b) Let $f(z)$ be an entire function whose image lies above the parabola $y = x^2$ in plane, i.e., $f$ maps the complex plane into $\{(x, y) : y > x^2\}$. Show that $f(z)$ must be a constant function.

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