Solution of Set 1

§ 1.1. 8. $$\frac{(8+2i)-(1-i)}{(2+i)^2} = \frac{7+3i}{3+4i} = \frac{(7+3i)(3-4i)}{(3+4i)(3-4i)} = \frac{33}{25} - \frac{19}{25}i$$

10. $$\left(\frac{2+i}{6i-(1-2i)}\right)^2 = \frac{3+4i}{(8i-1)^2} = \frac{3+4i}{-63-16i} = -\frac{(3+4i)(63-16i)}{(63+16i)(63-16i)}$$

$$= -\frac{253}{4225} - \frac{204}{4225}i$$

20 (c) $(2-i)z + 8z^2 = z(2-i+8z)$

$\therefore\ z_1 = 0,\ z_2 = -\frac{1}{4} + \frac{i}{8}$

(d) $z^2 + 16 = 0,\ \Rightarrow z^2 = -16,\quad \therefore\ z = \pm 4i$

24. Let $z = x+iy,\quad x, y \in \mathbb{R}$.

$$\operatorname{Im}\frac{1}{z} = \operatorname{Im}\frac{1}{x+iy} = \operatorname{Im}\frac{x-iy}{x^2+y^2} = -\frac{y}{x^2+y^2}$$

Since $y = \operatorname{Im} z > 0,\quad \frac{y}{x^2+y^2} > 0 \quad \therefore\ \operatorname{Im}\frac{1}{z} < 0$.

§ 1.2 6. Sol 1 $|(3+i)-6|^2 = |-3+i|^2 = 10$

$|(3+i)-(4+4i)|^2 = |-1-3i|^2 = 10$

$|(4+4i)-6|^2 = |4i-2|^2 = 20$

By the Pythagorean theorem (converse), $3+i,\ 6,\ 4+4i$ are the vertices of a right triangle.

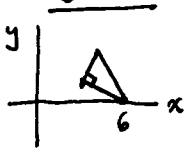
16. Let $z = x+iy,\quad x, y \in \mathbb{R}$.

$$\operatorname{Re}\frac{1}{1-z} = \operatorname{Re}\frac{1}{(1-x)-iy} = \operatorname{Re}\frac{(1-x)+iy}{(1-x)^2+y^2} = \frac{1-x}{(1-x)^2+y^2}$$

$$= \frac{1-x}{1+x^2+y^2-2x} = \frac{1-x}{1+1-2x} = \frac{1}{2}$$

where we used $x^2+y^2 = |z|^2 = 1$.

Sol 2 The 3 points in $\mathbb{R}^2$ are:



$(3,1),\ (6,0),\ (4,4)$.

$((6,0)-(3,1)) \cdot ((4,4)-(3,1))$ inner product.

$= (3,-1)\cdot(1,3) = 0$

§ 1.3 7 (d) $-2\sqrt{3}-2i = 4\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right) = 4e^{i\left(-\frac{5\pi}{6}\right)}$

$= 4\left(\cos\frac{-5\pi}{6} - i\sin\frac{5\pi}{6}\right)$

argument : $-5\pi/6 + 2k\pi, \quad k \in \mathbb{Z}$

(g) $\frac{-1+\sqrt{3}i}{2+2i} = \left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\cdot\frac{1}{1+i}$

$= e^{\frac{2\pi}{3}i}\cdot\frac{1}{\sqrt{2}\,e^{i\frac{\pi}{4}}}$

$= \frac{1}{\sqrt{2}}\,e^{\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)i}$

$= \frac{1}{\sqrt{2}}\,e^{\frac{5\pi}{12}i} = \frac{1}{\sqrt{2}}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$

argument : $\frac{5\pi}{12} + 2k\pi, \quad k \in \mathbb{Z}.$

10. (a) $\arg z_1z_2z_3 = \arg(z_1z_2)z_3 = \arg z_1z_2 + \arg z_3 = \arg z_1 + \arg z_2 + \arg z_3$

(b) $\arg z_1\bar{z}_2 = \arg z_1 + \arg\bar{z}_2 = \arg z_1 - \arg z_2$

12. (a) $\mathrm{Arg}(-6-6i) = -\frac{3\pi}{4}.$

(b) $\mathrm{Arg}(-\pi) = \pi$

(c) $\mathrm{Arg}(10i) = \pi/2$

(d) $\mathrm{Arg}(\sqrt{3}-i) = -\pi/6.$

- $\arg z = \mathrm{Arg}\, z + 2k\pi, \quad k \in \mathbb{Z}.$

$\arg\bar{z} = -\arg z = -\mathrm{Arg}\, z + 2k'\pi, \quad k' \in \mathbb{Z}.$

If z is not in $\mathbb{R}$, then $\mathrm{Arg}\, z \in (-\pi, \pi)$

$\therefore -\mathrm{Arg}\, z \in (-\pi, \pi) \quad \therefore \mathrm{Arg}\,\bar{z} = -\mathrm{Arg}\, z$

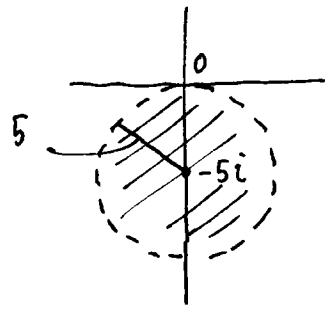
(a) If $z > 0$, $\mathrm{Arg}\, z = 0$. $\bar{z} = z$, $\mathrm{Arg}\,\bar{z} = 0$. $\therefore \mathrm{Arg}\,\bar{z} = -\mathrm{Arg}\, z$

(b) If $z < 0$, $\mathrm{Arg}\, z = \pi$. $\bar{z} = z$, $\therefore \mathrm{Arg}\,\bar{z} = \pi$ $\therefore \mathrm{Arg}\,\bar{z} \neq -\mathrm{Arg}\, z$

a) $|z+5i| < |4+3i|$.

Note $|4+3i| = 5$

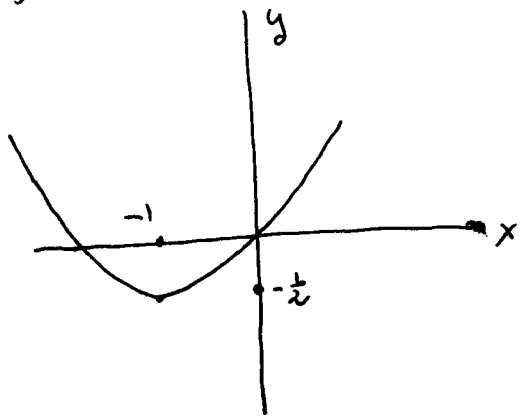
$\therefore |z+5i| < 5$.



b) $|z+1| = (\operatorname{Im} z) + 1$.

Let $z = x+yi$. Then

$|(x+1)+iy| = y+1$

$\therefore (x+1)^2 + y^2 = (y+1)^2$

$\therefore (x+1)^2 = 2y+1$ $\quad \therefore y = \frac{1}{2}(x+1)^2 - \frac{1}{2}$