Section 2.2, #3:

If $z_n \to z_0$ as $n \to \infty$, then for any $\epsilon > 0$ there is $N > 0$ s.t. for any $n > N$ we have $|z_n - z_0| < \epsilon$. Note

$$|x_n - x_0| \leq \sqrt{(x_n - x_0)^2 + (y_n - y_0)^2} = |z_n - z_0|$$

So $|x_n - x_0| < \epsilon$ if $n > N$. This means $x_n \to x_0$ as $n \to \infty$. Similarly, $y_n \to y_0$ as $n \to \infty$.

Conversely, if $x_n \to x_0$ and $y_n \to y_0$, then for any $\epsilon > 0$, there exists $N_1 > 0$ s.t. for any $n > N_1$ it holds $|x_n - x_0| < \epsilon/2$, and there is $N_2 > 0$ s.t. for any $n > N_2$ it holds $|y_n - y_0| < \epsilon/2$. Thus, if for any $n > N$ where N is the bigger one of N_1 and N_2, we have

$$|z_n - z_0| = \sqrt{(x_n - x_0)^2 + (y_n - y_0)^2} \leq |x_n - x_0| + |y_n - y_0| < \epsilon/2 + \epsilon/2 = \epsilon.$$

So $z_n - z_0$ as $n \to \infty$.