Math 300 Midterm Review

The midterm will be held in class on Thursday, February 11. Calculators, laptops, notes, "cheat sheets," etc. will NOT be allowed. The duration of the exam is 50 minutes.

The midterm will cover the material from Chapters 1, 2 and 3 of the text (but not including §1.7, 2.6, 2.7, 3.4, 3.6). The partial fractions, hyperbolic functions and the inverse trigonometric functions will NOT be tested on this midterm exam. The best ways to prepare for the midterm are to review your class notes, re-read the text material, review the homework problems, and work on other problems in the text. Below is a review of some the main ideas in each Chapter followed by key points to remember.

1. BASICS OF COMPLEX NUMBERS:

The set of complex numbers is the set $\mathbb{C} = \{a + ib\}$ where a, b are real numbers. We defined complex addition and multiplication on this set, under which the basic rules of arithmetic were obeyed. We viewed $\mathbb{C}$ as the 2 dimensional plane, and this gave a geometric context to the complex numbers, and to complex arithmetic in particular.

(1) Know the definition of complex numbers, and their cartesian and polar representations ($x+iy$ and $re^{i\theta}$). Be comfortable with complex arithmetic, in both its cartesian and polar form. In particular, understand what the *argument* of a complex number is. Understand what the branches $arg_{\tau} z$ are.
(2) Recall the definitions of the *modulus* and *conjugate* of a complex number. Know the geometric significance of these, and also how to use these in calculation.
(3) Recall Complex powers and roots. Know the geometric significance of taking the complex power or root of a complex number. Know De moivre's Theorem (expressed in either cartesian or polar form).
(4) Know the basic definitions for planar sets: (open set, closed set, connected set, domain). Be familiar with the use and meaning of dotted lines, open points...etc in sketching sets in the plane.

2. ANALYTIC FUNCTIONS IN GENERAL:

By a complex function $f(z)$ we just mean *any* function from some subset $S \subset \mathbb{C}$ to $\mathbb{C}$, which can be represented either as $f(x + iy) = u(x, y) + iv(x, y)$, or in terms of z, $\bar{z}$. In this chapter we gave definitions for when a complex function is *continuous* and when it is *(complex) differentiable* at a given point. We observed that the basic computation rules of differentiation from calculus (e.g. product rule, quotient rule, chain rule) are obeyed. We observed that while continuity of f simply meant continuity of u and v, differentiability of f was much more restrictive than u, v being differentiable in the usual real sense.

It was then shown that, up to continuity conditions on partial derivatives, complex differentiability corresponds to u and v satisfying the *Cauchy-Riemann equations* (Theorems 4 and 5, pp.73-74), which gives a convenient way to determine when/where a given function is differentiable. Finally, we discussed harmonic functions and showed that the real and imaginary parts of an analytic function were *harmonic conjugates.*

(1) Know the basic definition for $f(z)$ to be *differentiable* at a point z_0, *analytic* at a point z_0 and *analytic* on a set $S \subset \mathbb{C}$.
(2) Know the *Cauchy-Riemann equations* and how they are used in determining differentiability and analyticity.
(3) Know the definition of a harmonic function, and the relation between harmonic functions and analytic functions. In particular, given a harmonic function, know how to find a harmonic conjugate.

3. THE ELEMENTARY ANALYTIC FUNCTIONS:

The most important elementary analytic functions are: *polynomials, rational functions, exponential, trigonometric functions, logarithm and power functions.* Note that except for the polynomials and rational functions, the elementary analytic functions were defined using either the complex exponential e^z, or the complex logarithm $\log z$.

(1) Know the definitions of the basic analytic functions, their basic properties, their domains of analyticity, and their derivatives. Know the meaning of zeros of polynomials and poles of rational functions.
(2) Understand the multi-valued-ness of $\log z$ and z^α.
(3) Know what the standard branches of $\log z$ are, and their domains of analyticity. Know the corresponding branches of the power function z^α.
(4) Reminder: for any branch $\log_\tau z$, we have $e^{\log_\tau z} = z$ for all z. On the other hand, $\log_\tau(e^z) = z + i2\pi k$ for some integer k.
(5) Reminder: for any branch $\log_\tau z$, it is not generally true that $\log_\tau(z_1 z_2) = \log_\tau z_1 + \log_\tau z_2$.
(6) Reminder: some, but not all of the laws of exponents were true involving complex exponentiation: For example, although we always have
$$e^{z_1} e^{z_2} = e^{z_1 + z_2},$$
$(e^{z_1})^{z_2}$ does not equal $e^{z_1 z_2}$ in general. Similarly, although we always have
$$z^\alpha z^\beta = z^{\alpha+\beta}$$
provided the same branch is used for each power, $(z^\alpha)^\beta$ does not equal $z^{\alpha\beta}$ in general.
(7) Understand the multi-valued-ness of $\log z$ and z^α.
(8) Know what the standard branches of $\log z$ are, and their domains of analyticity. Know the corresponding branches of the power function z^α.
(9) Know how to determine the domains of analyticity for compositions of the elementary analytic functions.
(10) In general, know what it means for a function to be a branch of a multi-valued complex function.