Problem 1 (9 points) Write down the negation of each of the following statement.

(a) $\forall x > 1$, $\exists y \in \mathbb{R}$ such that $2x + y < 0$.

(b) $\exists x \in [0, 1)$ such that $f(x) > 1$ or $f(x) \leq -1$.

(c) $x \in A$ and $x \notin B$

Problem 2 (18 points) Let A, B, C be three sets. Let P be the statement :

$$A \subseteq B \implies A \cap C \subseteq B \cap C.$$

(a) Is P true or false? Justify your answer.

(b) Write down the converse of P. Is the converse true or False? Justify your answer.

(c) Write down the inverse of P. Is the inverse True or False? Justify your answer.

Problem 3. (10 points) Let $f : \mathbb{N} \to \mathbb{Z}$ be a function defined by: for $n \in \mathbb{N}$

$$\begin{aligned} f(2n) &= n^2 \\ f(2n-1) &= -5n. \end{aligned}$$

(a) [3 points] Is f surjective? Justify your answer.

(b) [7 points] Is f injective? Justify your answer.

Problem 4. (13 points)

(a) [3 points] Define what it means for a set A to be countable?

(b) [5 points] Let $f : A \to \mathbb{N}, g : \mathbb{N} \to B$ be functions. If $g \circ f$ is injective, show that A is countable.

(c) [5 points] Let S be a denumerable set. Prove or disprove the statement: there are only finitely many denumerable subsets of S.