HW Set 1 Solution

1.2 (b) True. Truth table for "$p \Rightarrow q$"

(d) True. $\sim(p \wedge q) \Leftrightarrow (\sim p) \vee (\sim q)$ (Practice 1.6 (a), p. 6)

(e) False. $\sim(p \Rightarrow q)$ is $p \wedge \sim q$

1.4. (c) The function f is not injective or it is not surjective

(d) $x \geq 5$ and $x \leq 7$. (or : $5 \leq x \leq 7$).

(e) $x > 3$ and $f(x) \leq 7$

1.6. (a) Antecedent: $5n$ is odd, Consequent: n is odd

(b) " : it is monotone and bounded " : a sequence is convergent

(c) " : it is convergent " : a real sequence is Cauchy

(d) " : convergence " : boundedness

1.8 (a)

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

(b)

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

(c)

p	q	$\sim q$	$p \Rightarrow q$	$\sim q \wedge (p \Rightarrow q)$	$[\sim q \wedge (p \Rightarrow q)] \Rightarrow \sim p$	$\sim p$
T	T	F	F	F	T	F
T	F	T	T	T	F	F
F	T	F	T	F	T	T
F	F	T	T	T	T	T

1.10 (b) T (c) F (e) T

2.4 (c) There exists a bounded interval that contains infinitely many integers.

(d) $\forall x$ in S, $x < 5$.

(e) $\exists x$, $0 < x < 1$, $f(x) \geq 2$ and $f(x) \leq 5$.

(f) $x > 5$ and $\forall y > 0$, $x^2 \leq 25 + y$

2.6 (a) T (b) F (c) T (d) F (e) F (f) T

(a) take $z = x+y$

(b) take $x=0$, $z=y+1$

(c) take $x=1$, $z=y$

(d) take $x=1$, $y=0$

(e) let $x=2$. For any given y, let $z=y+1$. $\therefore$ $z>y$ holds but "$z > x+y$" is false.

(f) Given any x and y, choose $z = x+y+1 > x+y$ or choose $z = y-1 < y$

2.8 (c)

2.12 (a) $\exists k > 0$, $\underbrace{\text{such that}}^{\ni}$ $\forall x$, $f(x+k) = f(x)$

(b) $\forall k > 0$, $\exists x$, such that $f(x+k) \neq f(x)$

2.14 (a) $\forall x$ and $\forall y$, $x < y \Rightarrow f(x) > f(y)$

(b) $\exists x$ and $\exists y$, such that $x < y$ and $f(x) \leq f(y)$

2.15 (a) $\forall x$ and $\forall y$, $f(x) = f(y) \Rightarrow x = y$

(b) $\exists x$ and $\exists y$, $f(x) = f(y)$ and $x \neq y$

2.16 (a) $\forall y$ in B, $\exists x$ in A, such that $f(x) = y$

(b) $\exists y$ in B, such that $\forall x$ in A, $f(x) \neq y$