Practice Midterm Exam Math 220

Problem 1. Use Mathematics Induction to show: $\forall n \geq 4, n \in \mathbb{N}$, $n! \geq n^2$. (Recall: $n! = 1 \cdot 2 \cdot \ldots \cdot n$)

Problem 2. For each set S below, determine:

(1) $\text{int}S$, (2) $\text{bd}S$, (3) $\max S$, (4) $\sup S$, (5) the set S' of the accumulation points of S .

(a) $S = [-1, 2) \cup (2, 3)$

(b) $S = \displaystyle\bigcup_{n=2}^{\infty} \left[-\frac{1}{n}, \frac{2}{n}\right)$

(c) $S = \left\{1 + \dfrac{(-1)^n}{n} : n \in \mathbb{N}\right\}$

(d) $S = \left\{p \in \mathbb{R} \backslash \mathbb{Q} : 2 < p < 2\sqrt{2}\right\}$

Problem 3. Mark "True" or "False" to each of the following statement. If true, provide a proof; if false, provide a counterexample.

(a) Let S, T be subsets of $\mathbb{R}$. If $S \subseteq T$, then the set of accumulation points of S is contained in the set of accumulation points of T.

(b) Let S be a subset of $\mathbb{R}$ and let a be a lower bound of S. If $a \in S$, then S is not open.

(c) Let S, T be subsets in $[0, 1)$ and $\sup T \in [0, 1)$. Then $\sup \left\{ \frac{s}{1-t} : s \in S, t \in T \right\}$ exists in $\mathbb{R}$.

Problem 4. Let $S_1, S_2, ..., S_n$ be subsets of $\mathbb{R}$. Prove that

$$\operatorname{int}(S_1 \cap S_2 \cap ... \cap S_n) = (\operatorname{int} S_1) \cap (\operatorname{int} S_2) \cap ... \cap (\operatorname{int} S_n).$$