HW set 2 Solution

Section 3:

3.2 (a) True: comment in first paragraph on page 18.
(b) False: it's called a contradiction.
(c) True: comment after Practice 3.8.
(d) True: end of Example 3.1.
(e) False: must show $p(n)$ is true for all n.

3.4 (a) If all violets are blue, then all roses are red.
(b) If H is normal, then H is not regular.
(c) If K is compact, then K is closed and bounded.

3.6 (a) Any real number x with $x<-2$, say $x=-3$.

(e) 2.

(f) $n=1$ or any odd n.

(g) $x=2$ and $y=18$, or $x=3$ and $y=12$.

(l) There is only one counterexample: $x=0$. In fact $x^3+(x-1)^2=x^2+1 \Rightarrow x^3-2x=0 \Rightarrow x=0$ or $x=\pm\sqrt{2}$. Since $\pm\sqrt{2}$ are not rational numbers, $x=0$ is the unique rational number such that $x^3+(x-1)^2=x^2+1$.

3.8 Suppose $f(x_1)=f(x_2)$. That is, $3x_1-5=3x_2-5$. Then $3x_1=3x_2$, so $x_1=x_2$.

Section 4.

4.4 $x^2+\frac{3x}{2}=1 \Rightarrow 2x^2+3x-2=0 \Rightarrow x=\frac{-3\pm\sqrt{3^2-4\cdot2(-2)}}{4}=\frac{-3\pm5}{4}$

$\Rightarrow x=-2$ or $x=\frac{1}{2}$. Hence, the rational number x such that $x^2+\frac{3x}{2}=1$ is NOT unique.

4.6 Solve $t^2 - 6xt + 9 = 0$ for t (4)

to obtain $t = 3x \pm 3\sqrt{x^2-1}$.

Let $y = 3x + 3\sqrt{x^2-1}$ and $z = 3x - 3\sqrt{x^2-1}$, $x^2-1>0$ as $x>1$.

(or let $y = 3x - 3\sqrt{x^2-1}$ and $z = 3x + 3\sqrt{x^2-1}$). Then

$x = \frac{y^2+9}{6y} = \frac{z^2+9}{6z}$ and $y \neq z$ (because $x^2-1 \neq 0$).

4.9 Hint in book: Suppose $\log_2 7$ is rational and find a contradiction.
Proof: Suppose $\log_2 7 = a/b$, where a and b are integers. We may assume that $a > 0$ and $b > 0$. We have $2^{a/b} = 7$, which implies $2^a = 7^b$. But the number 2^a is even and the number 7^b is odd, a contradiction. Thus $\log_2 7$ must be irrational.

4.19 It is true. We may label the numbers as $n, n+1, n+2, n+3$, and $n+4$. We have the sum

$$S = n + (n+1) + (n+2) + (n+3) + (n+4) = 5n + 10 = 5(n+2),$$

so that S is divisible by 5.

4.22 False. Let $n = 1$. Then $n^2 + 4n + 8 = 13$. Any other odd n will also work.

4.25 Suppose $x > 0$. Then $(x+1)^2 = x^2 + 2x + 1 > x^2 + 1$, so the first inequality holds. For the second inequality, consider the difference $2(x^2+1) - (x+1)^2$. We have

$$2(x^2+1) - (x+1)^2 = 2x^2 + 2 - x^2 - 2x - 1 = x^2 - 2x + 1 = (x-1)^2 \geq 0.$$

for all x, so the second inequality also holds.