FINAL EXAM

Math 220 Section 101

December 4, 2008

Last Name: ____________________

First Name: ____________________

Student Number: ____________________

Signature: ____________________

The exam is worth a total of 100 points with duration 2.5 hours. No books, notes or calculators are allowed. Justify all answers, show all work and **explain your reasoning carefully**. You will be graded on the clarity of your explanations as well as the correctness of your answers.

UBC Rules governing examinations:

(1) Each candidate should be prepared to produce his/her library/AMS card upon request.
(2) No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.
(3) Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination, and and shall be liable to disciplinary action:
a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
b) Speaking or communicating with other candidates.
c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness will not be received.
(4) Smoking is not permitted during examinations.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total	

Problem 1 (15 points). Determine whether the following series converge or diverge. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n} + \sin n}{n(\sqrt{n} - 1)}$

Problem 2 (15 points). Prove the following limits exist and then evaluate them.

(a) $\lim_{n\to\infty} \left(\sqrt{9n^6 + 6n^3} - 3n^3\right)$

(b) $\lim_{n\to\infty} x_n$, where $x_1 = 2, x_{n+1} = \sqrt{3x_n + 4}$

Problem 3 (12 points) For each of the following statement, circle one answer and only one answer. You do not need to give reasons.

(a) The set of boundary points of the set $\bigcap_{n=1}^{\infty}(0, \frac{1}{n})$ is $\{0\}$.

Answer: True False

(b) If f and g are injective functions from $\mathbb{R}$ to $\mathbb{R}$, then $f \circ g$ and $g \circ f$ are both injective.

Answer: True False

(c) Let P be the statement: For all $x \in (0,1)$, $1 < f(x) < 10$. The negation of P is: there exists $x \in (-\infty, 1] \cup [1, \infty)$, such that $f(x) \leq 1$ or $f(x) \geq 10$.

Answer: True False

(d) If $a_n > 0$ and the sequence (a_n) is unbounded, then $(\frac{1}{a_n})$ converges to 0.

Answer: True False

(e) If (a_n) and (b_n) are both divergent sequences of real numbers, then the sequence $(a_n b_n)$ is also divergent.

Answer: True False

(f) The sequence (a_n) is given by: $a_1 = 8, a_n = (-1)^n \frac{5n}{n+1}$ for $n \geq 2$. Then $\limsup a_n$ is

Answer: (A) 8 (B) 5 (C) -5 (D) Does not exist

Problem 4 (25 points). Mark each statement True or False. If True, give a proof. If False, give a counter-example.

(a) Let $f : \mathbb{N} \to \mathbb{N}$ and $g : \mathbb{N} \to \mathbb{N}$ be functions. Then $f + g : \mathbb{N} \to \mathbb{N}$ is not surjective.

(b) Let A and B be two sets and $A \subseteq B$. If A is denumerable and B is uncountable, then $B \backslash A$ is uncountable.

(c) Let x and y be two given real numbers. If $x < y\epsilon$ for any $\epsilon > 0$ and $y \geq 0$, then $x \leq 0$.

(d) Let (a_n) be a sequence of real numbers and $a \in \mathbb{R}$. Suppose that for any positive rational number $r \in \mathbb{Q}$, there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0$, $|a_n - a| < r$. Then $\lim_{n\to\infty} a_n = a$.

(e) Let $S = (0, 1) \cap \mathbb{Q}$, then S is neither open nor closed.

Problem 5 (8 points) Let A and B be subsets of the interval $(0, 1)$ with $\sup A = \sup B = 1$. Let $C = \{ab : a \in A, b \in B\}$. Prove that $\sup C = 1$.

Problem 6 (10 points) Let $f : A \to B$ be a function and let S and T be subsets of A. If $f(S) = f(T)$ and f is injective, show $S = T$.

Problem 7 (15 points) Mark each statement True of False, if True, give a proof, if False, provide a counterexample.

(a) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.

(b) If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.