Math 220, Review

The final exam of Section 201 will be held at LSK Room 200, 3:30p.m. - 6:00p.m. on April 29, 2010 Thursday. Books, notes and "cheat sheets" will NOT be allowed.

The best ways to prepare for the final exam are to review your class notes, re-read the text material, review the homework problems, and work on other problems in the text. Below is a review of some of the key concepts and results to remember.

Chapter 1. Know the quantifiers ($\forall, \exists$), negations, converse, inverse of a statement, proof by contradiction, counterexample.

Chapter 2.

- Know the basic set operations (union, intersection, complement), subsets
- know how to prove two sets are equal (to show $A = B$, ones needs to show two inclusions: $A \subseteq B$ and $B \subseteq A$).
- Relations, equivalent relations, equivalent classes, Cartesian product.
- Functions:
  - The definition of a function being injective, surjective, bijective. The domain of f, the range of f and composition of functions.
  - For a function $f : A \to B$, know the meaning of the image set $f(C)$ for $C \subseteq A$ and the pre-image set $f^{-1}(D)$ for $D \subseteq B$.
  - Understand when the inverse function can be defined.
- Definition of a set is finite, infinite, denumberable, countable, uncountable. Important results:
  - Any subset of a countable set is countable.
  - A countable union of countable sets is countable.
  - For a non-empty set S, the following conditions are equivalent:
    1. (1) S is countable
    2. (2) There is an injection $f : S \to \mathbb{N}$
    3. (3) There is a surjection $g : \mathbb{N} \to S$.
  - $\mathbb{Q}$ is denumerable and $\mathbb{R}$ is uncountable.

Chapter 3.

- Well-ordering property of $\mathbb{N}$
- Mathematical Induction
- Absolute values of real numbers, the triangle inequality
- Upper bounds of S, $\max S$, $\sup S$, lower bounds of S, $\min S, \inf S$. Important: $\max S, \min S$ must be in S, but $\sup S, \inf S$ may or may not be in S.
- Completeness Axiom of $\mathbb{R}$. Some of its useful consequences:
  - The set $\mathbb{N}$ is unbounded in $\mathbb{R}$ (Archimedean property)
  - For each $x > 0$, there is an $n \in \mathbb{N}$ such that $0 < 1/n < x$
  - The equation $x^2 = p$ is solvable in $\mathbb{R}$ for each prime number p
  - Density of $\mathbb{Q}$ in $\mathbb{R}$ (and density of the irrational numbers in $\mathbb{R}$)

- Know the definitions of neighborhood of x, deleted neighborhood of x, interior points of S, boundary points of S, closed sets, open sets, accumulations points of S, isolated points of S, $\text{cl}\, S$. Important results:
  (1) S is closed (open) if and only if $\mathbb{R}\backslash S$ is open (closed)
  (2) S is open if and only if every x in S is an interior point of S
  (3) S is closed if and only if S contains all of its accumulation points
  (4) $\text{cl}\, S$ is closed
  (5) S is closed if and only if $S = \text{cl}\, S$
  (6) $\text{cl}\, S = S \cup \text{bd}\, S$
  (7) The union of any collection of open sets is open, the intersection of FINITELY many open sets is open
  (8) The intersection of any collection of closed sets is closed, the union of FINITELY many closed sets is closed

Chapter 4.

- Sequences, the definitions of (s_n) converges to s, diverges, bounded, diverge to $+\infty, -\infty$.
- Limit theorems.
- A convergent sequence is bounded.
- Monotone sequence (increasing, decreasing), Monotone convergence theorem.
- Definition of a Cauchy sequence, Cauchy Convergence Criterion: A sequence of real numbers is convergent if and only if it is a Cauchy sequence.
- Subsequences:
  - Bolzano-Weierstrass theorem for sequences: Every bounded sequence has a convergent subsequence.
  - Definition of $\limsup s_n, \liminf s_n$.
  - The set S of subsequential limits of (s_n).

Chapter 8.

- Infinite series (partial sum, convergent, divergent, diverge to $+\infty$)
- If a series $\sum a_n$ converges, then $\lim a_n = 0$.
- Cauchy criterion for series
- $\sum_{n=1}^{\infty} 1/n^p$ is convergent if $p > 1$ and is divergent if $p \leq 1$, when $p = 1$, the series is called the harmonic series.
- The Geometric series $\sum_{n=0}^{\infty} r^n$ converges to $1/(1-r)$ if and only if $|r| < 1$. It diverges if $|r| \geq 1$.
- Definition of a series converges conditionally, absolutely. If a series converges absolutely, then it converges.
- Convergent tests: (1) Comparison test (2) Ratio test (3) Root test (4) Alternating series test (note: a series is alternating if it can be written as $\sum(-1)^n a_n, a_n \geq 0$, to apply the test, one needs to know: $\lim a_n = 0$ and (a_n) is decreasing).