Set 1, Due: January 28, 2020

1. p. 47: 2-1.
2. p. 48: 2-9, 2-.10 (read the definition of gradient of a function, p. 27)
3. Let $\nabla$ be an affine connection on an $n$-dimensional manifold $M$. Let $T : TM \times \cdots \times TM \to C^\infty(M)$ be a $(r,0)$-type tensor. The covariant derivative $\nabla T$ is a tensor of $r + 1, 0$-type defined by
   \[
   \nabla T(X_1, \ldots, X_r, X) = X(T(X_1, \ldots, X_r)) - T(\nabla_X X_1, \ldots, X_r) - \cdots - T(X_1, \ldots, \nabla_X X_r).
   \]
   For $X \in \Gamma(TM)$, the covariant derivative $\nabla_X T$ of $T$ relative to $X$ is a $(r,0)$-type tensor given by
   \[
   \nabla_X T(X_1, \ldots, X_r) = \nabla T(X_1, \ldots, X_r, X).
   \]
   Let $g$ be a Riemannian metric on $M$ and $\nabla$ be the Levi-Civita connection of $(M, g)$. Show $\nabla g = 0$.
4. Consider two parametrizations of the 2-dim torus $T^2$:
   (a) $F_1(\alpha, \beta) = (e^{\sqrt{-1} \alpha}, e^{\sqrt{-1} \beta}) \subset \mathbb{R}^4$
   (b) $F_2(\alpha, \beta) = ((2 + \cos \alpha) \cos \beta, (2 + \cos \alpha) \sin \beta, \sin \alpha) \subset \mathbb{R}^3$.
   Hence $T^2$ is equipped with two Riemannian metrics via the pullbacks by $F_1, F_2$. In each case, compute $[\frac{\partial}{\partial \alpha}, \frac{\partial}{\partial \beta}], \nabla \frac{\partial}{\partial \alpha}, \frac{\partial}{\partial \beta}$ where $\nabla$ is the Levi-Civita connection in each case.
5. p.49: 2-15
6. p.50: 2-18