Set 4 Due on November 2, 2017

- (a) Show, by direct computation, that for $f : R^2 \to R$ defined by $f(x, y) = x^2$ that
$$\frac{\partial f}{\partial x}\frac{\partial}{\partial x} + \frac{\partial f}{\partial y}\frac{\partial}{\partial y} \neq \frac{\partial f}{\partial r}\frac{\partial}{\partial r} + \frac{\partial f}{\partial \theta}\frac{\partial}{\partial \theta} \tag{1}$$
but
$$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{\partial f}{\partial r}dr + \frac{\partial f}{\partial \theta}d\theta.$$
(b) The euclidean inner product is
in (x, y) coordinate: $g = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
in (r, θ) coordinate: $g = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$. Let g^{ij} be the (i, j)-entry of the inverse matrix g^{-1}. Show, by direct computation, that
$$g^{ij}\frac{\partial f}{\partial x^i}\frac{\partial}{\partial x^j}$$
is independent of the two coordinate systems.
- page 201: 8-10, 8-16
- Use local coordinate charts, prove
(a) the Jacobi identity for the Lie bracket.
(b) For a diffeomorphism $f : M \to N$, $f_*[X, Y] = [f_*X, f_*Y]$ for smooth vector fields X, Y on M.
- page 199: 8-1 (review the extension result for smooth functions, page 45)