Homework Set 3, Math 424
Due: February 26, 2014

(1) Let $S(r) = \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 + z^2 = r^2 > 0\}$ and N be the outward pointing unit normal field on $S(r)$. Find the eigenvalues of $dN|_p, p \in S(r)$. Compare the 2nd fundamental forms of $S(r)$ and $\mathbb{S}^2$, both for the outward pointing unit normal.

(2) Let $C(r) = \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 = r^2 > 0\}$ be the right cylinder and N be the outward pointing unit normal field on C. Find the eigenvalues of $dN|_p, p \in C(r)$.

(3) Let S be the surface defined by $z = y^2 + ax^2$ in $\mathbb{R}^3$ for some nonzero constant a. Find the eigenvalues of $dN|_{(0,0,0)}$, where $N((0, 0, 0)) = (0, 0, 1)$.

(4) Show that if a regular surface is tangent to a plane along a curve, then the points of this curve are either parabolic or planar.

(5) Let $C \subset S$ be a regular curve on a regular surface S with Gauss curvature $K > 0$. Show that the curvature k of C at $p \in C$ satisfies

$$k \geq \min\{|k_1(p)|, |k_2(p)|\}$$

where $k_1(p), k_2(p)$ are the principal curvatures of S at p.

(6) Let $f : U \subset \mathbb{R}^3 \to \mathbb{R}$ be a differentiable function and a is a regular value of f. Show that the regular surface $S = \{(x, y, z) \in \mathbb{R}^3 | f(x, y, z) = a\}$ is orientable. (hint: by considering curves in S through an arbitrary point $p \in S$, construct a unit normal vector field on S)