1. Prove that the set of all irrational numbers is uncountable.
2. Prove or disprove: If $A \subseteq B \subseteq C$ and $A$ and $C$ are countably infinite, then $B$ is countably infinite.
3. Prove or disprove: There exists a bijective function $f : \mathbb{Q} \rightarrow \mathbb{R}$.
4. Prove or disprove: The set $\mathbb{Z} \times \mathbb{Q}$ is countably infinite.
5. Prove or disprove: The set $\{(a_1, a_2, a_3, \ldots) : a_i \in \mathbb{Z}\}$ of infinite sequences of integers is countably infinite.
6. Describe a partition of $\mathbb{N}$ that divides $\mathbb{N}$ into $\aleph_0$ countably infinite subsets.
7. Suppose $A = \{(m, n) \in \mathbb{N} \times \mathbb{R} : n = \pi m\}$. Is it true that $|\mathbb{N}| = |A|$?
8. Let $F_n = \{X \subset \mathbb{N} : |X| = n\} \subseteq \mathcal{P}(\mathbb{N})$.
   Prove that for every $n \in \mathbb{N}$, $|F_n| = |\mathbb{N}|$. Also show that $\left| \bigcup_{n \in \mathbb{N}} F_n \right| = |\mathbb{N}|$. Does this result contradict with the fact that $|\mathbb{N}| < |\mathcal{P}(\mathbb{N})|$, and why?
9. Show that the two given sets have equal cardinality by describing a bijection from one to the other.
   Describe your bijection with a formula (not as a table).
   (a) $\mathbb{R}$ and $(\sqrt{2}, \infty)$
   (b) The set of even integers and the set of odd integers
   (c) $\mathbb{Z}$ and $S = \{x \in \mathbb{R} : \sin x = 1\}$
   (d) $\{0, 1\} \times \mathbb{N}$ and $\mathbb{Z}$
10. Define $P$ to be the set of all polynomials with rational coefficients. That is,
    
    $$P = \left\{ a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n : n \in \mathbb{N} \text{ and } a_i \in \mathbb{Q} \text{ for all } i \in \{0, 1, \ldots, n\} \right\}.$$
    (a) Prove or disprove: $P$ is countable.
    (b) Define $A$ to be the set of all real numbers that are the roots of a polynomial in $P$. That is,
    
    $$A = \{x \in \mathbb{R} : \exists f \in P - \{0\} \text{ s.t. } f(x) = 0\}.$$
    Prove or disprove: $|A| = |P|$.
11. Let $A$ and $B$ be sets. Let $P$ be a partition of $A$, and let $Q$ be a partition of $B$. Suppose that $h : P \rightarrow Q$ is a bijection. Suppose also that for each $X \in P$, the sets $X$ and $h(X)$ have the same cardinality. Prove that $A$ and $B$ have the same cardinality.